THE INDUCTION MOTOR

Published by the

McGraw-Hill Book Company

New York

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McGraw Publishing Company   Hill Publishing Company

Publishers of Books for

Electrical World   The Engineering and Mining Journal

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THE
INDUCTION MOTOR

BY

BENJ. F. BAILEY, B.S., PH.D.
Junior Professor of Electrical Engineering, University of Michigan

McGRAW-HILL BOOK COMPANY
239 WEST 39TH STREET, NEW YORK
6 BOUVERIE STREET, LONDON, E.C.
1911

PREFACE

In presenting a new treatise on the induction motor, the writer is aware that he is entering a field in which there are already many excellent works. In this book, however, an attempt is made to present the subject from a somewhat new standpoint. The endeavor has been to produce a work that will have the greatest possible value for those who wish to inform themselves more fully regarding the theory of the induction motor than they can by studying the elementary text-books, but who at the same time do not care to go too deeply into the theoretical aspects of the question. The writer's aim has, therefore, been to present so much of the theory as is necessary to understand the phenomena of the induction motor, so far as these phenomena relate to the design or operation of these machines.

The student is assumed to have some knowledge of alternating currents, and to understand in a general way the operation of the alternator, the synchronous motor, the induction motor, etc. To this end, he is supposed to have read some of the several elementary texts dealing with these subjects.

Throughout the book, an earnest endeavor has been made to present clearly the physical conception of the actions taking place. It is the writer's belief that nine-tenths of the trouble experienced by many people in studying the action of electrical machinery comes from the lack of a clear idea of the elementary physical actions. An attempt is made to apply mathematical reasoning to the problem before this understanding is obtained, and the result is a mental haze, which is perhaps never dissipated. The reader is therefore strongly urged to study carefully the first two chapters, and make sure that they are fully understood, before going farther.

Several subjects of the greatest practical importance have been only briefly mentioned, if treated at all, by previous writers. Some of these are the variation of the starting torque in different positions of a wound rotor, the disadvantage of too great starting torque in squirrel-cage motors, the iron losses in the rotor teeth, etc. It is also thought that the examples of design given in most of the books

on this subject are not representative of present conditions. This is on account of the fact, well known to designers, that in recent years a marked reduction in the weight and dimensions of induction motors has taken place. It is believed that the examples of design given are fairly representative of average modern motors.

The writer wishes to take this opportunity to acknowledge his indebtedness to those who have preceded him in this field. Every work of an engineering nature is necessarily founded on that of others. An attempt to give original demonstrations of all of the elementary facts in relation to a subject, necessarily leads to a far less simple treatment than the frank use of older methods, when these methods are at least as clear as anything the author has to offer in their stead. In this regard, the present treatise is no exception. Particular mention should be made of the excellent works of Behrend, Boy de la Tour, and McAllister. In particular, the treatise of the last named author has been drawn upon for several simple and lucid demonstrations, notably for the proof of the circle diagram, and for one of the methods of treating the subject of single-phase motors. Credit has been given in the text for contributions of various writers. When this has not been done, the omission is due either to a lack of knowledge on the part of the author, of the work in question, or an uncertainty regarding the author to whom such credit should be given.

Lack of time and space has caused the omission of much material of historical interest. The same reason has caused the omission of the discussion of many ingenious devices designed to improve in various ways the performance of induction motors. Many of these relate to methods of improving the starting performance of induction motors, and present numerous points of theoretical and practical importance. In order to keep the size of the book within reasonable limits, it was thought best to limit the discussion almost entirely to devices in successful operation, and to those of recent introduction which seem most likely to be of permanent value. For the same reason, the treatment has been almost entirely confined to motors built in the United States.

For much of the experimental work mentioned in the text the author is indebted to the careful work of Mr. H. L. Tanner, Instructor in the Department of Electrical Engineering of the University of Michigan, and to Mr. Stanley D. Livingstone, assistant in the same.

Ann Arbor, Mich., September, 1911.

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THE INDUCTION MOTOR

CHAPTER I

ELEMENTARY THEORY

An induction motor consists essentially of two parts—a stationary part, commonly called the stator, and a rotating part, called the rotor. The terms primary and secondary are sometimes used instead of the above. These expressions are not strictly interchangeable, since the term primary refers to the part to which the current is supplied, and the

Fig. 1.—Fairbanks Morse Induction Motor.

term secondary to the part in which the current is induced. Usually the stator is the primary, but in some cases this is reversed and the rotor becomes the primary. Some writers use the terms field and armature, from analogy with direct-current practice. This, however, is open to serious objection, since, as will appear presently, the primary is the part which corresponds in function to the armature of a direct-current machine, so that what is thus called the field is in reality the armature.

Figs. 1 and 2 show respectively a complete squirrel-cage induction motor, and the same motor unassembled. The active iron of both the

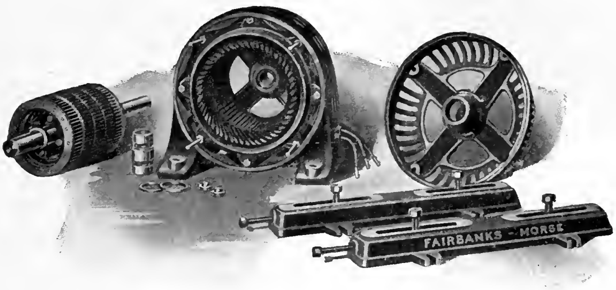


Fig. 2.—Unassembled Parts of Fairbanks Morse Induction Motor.

stator and the rotor consists of thin laminations, provided with slots around the periphery, and held together by suitable means to form a rigid structure. Fig. 3 is a drawing of a pair of typical stator and rotor

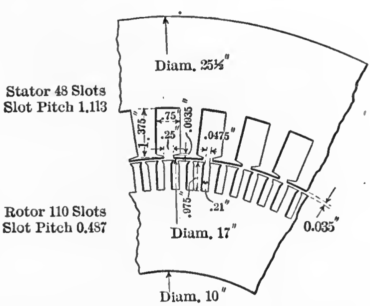


Fig. 3.—Stator and Rotor Laminations.

laminations. The windings of the stator and rotor are placed in the slots of the core. In many cases the winding of the rotor consists of copper bars placed in the slots, and short circuited at both ends by heavy copper or brass rings.

The stator and rotor coils are connected in the same manner as

though the machine were to be used as an alternating-current generator. Any of the well-known windings which can be used for an alternator can be employed in the induction motor. A few points of difference will be brought out later. For the present, it is sufficient to point out that the use of short, or fractional-pitch windings, is more common in the case of induction motors than in the case of alternators.

If two-phase or three-phase current be supplied to a stator properly wound for the corresponding number of phases, a rotating magnetic field will be set up. This field will be nearly uniform, and will rotate with practically uniform velocity. Its speed will be such that it will move the distance between two poles of the stator in the time required for the current to complete one-half cycle. This flux, cutting through the con-

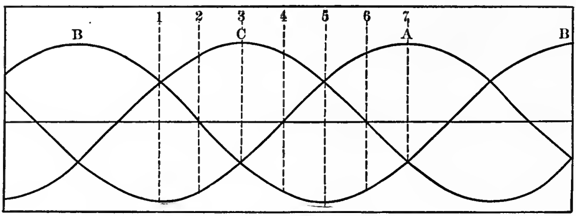


FIG. 4.—Three-phase Current Waves.

ductors of the rotor, produces currents in them. These currents are acted upon by the stator magnetism, and a torque is set up which causes the rotor to revolve.

The following will explain how this takes place in a three-phase winding. In Fig. 4 are shown the three currents of a three-phase circuit. They differ in phase, as shown by one-third of a cycle, or 120 electrical degrees. Fig. 5 shows a section of the stator and rotor of an induction motor. Only about two poles are shown, but it will be understood that the motor might have any number of poles. The number of slots in this case is taken as six per pole or two per phase per pole. We might use any number from one up, although less than two or more than eight are rarely employed. The winding is supposed to be a full-pitch winding, that is, the coils span a complete pole pitch, or from slot 1 to 7, 2 to 9, etc. For our present purpose the end connections are immaterial. The only requirement is that the coils be so connected that those lying in the slots marked A are all connected in one circuit and have the current A in

them. Likewise those coils in slots B are connected in the B circuit, and those in C in the C circuit. It should not be inferred that a slot always has current of only one phase in it. It is always true when we employ full-pitch windings, but is not the case with short-pitch windings.

Consider the currents in the stator at the time marked 1 on Fig. 4.

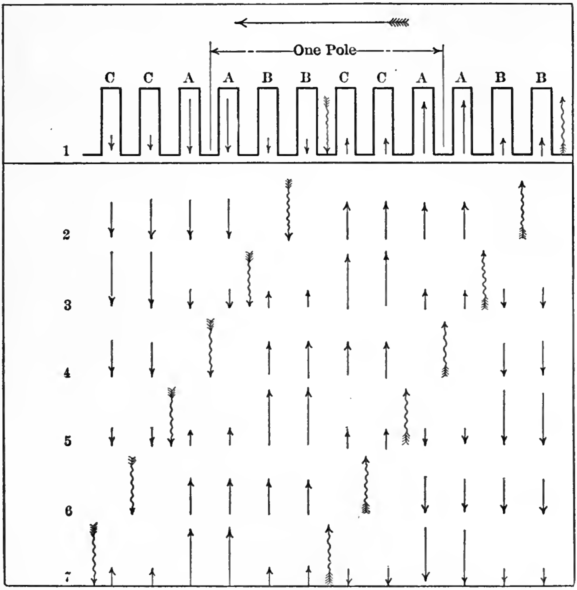


FIG. 5.—Distribution of Currents in Stator of a Three-phase Induction Motor.

The current A may be regarded as negative, and both B and C as positive. The relative values of these currents are represented below the slots of Fig. 5 by arrows, whose lengths are proportional to the strength of the current and whose directions correspond to the direction of the current.

Between the slots B and C will be seen a waved arrow. This indicates the point of greatest magnetic flux, and its direction shows the direction of the flux. The waved arrows then show the centers of the poles of the stator at any particular instant. The maximum flux is

obviously located at the point shown by the arrows, since all the currents to the left of it are in one direction, and all those to the right are in the opposite direction.

At the time marked 2 in Fig. 4, the current *A* has decreased somewhat, while *C* has increased, and *B* has dropped to zero. Both *A* and *C* retain the same direction. The values of the currents in the various slots are as shown by the arrows opposite the number 2. The position of the maximum flux is obviously in the position indicated.

Likewise, the state of the flux at the times 3, 4, 5, etc., are as shown opposite the numbers 3, 4, 5. It will be seen that the arrow indicating the position of maximum flux moves steadily to the left. Each increment of time considered is that corresponding to 30 degrees. Since there are twelve slots per pair of poles, and since the flux moves one slot

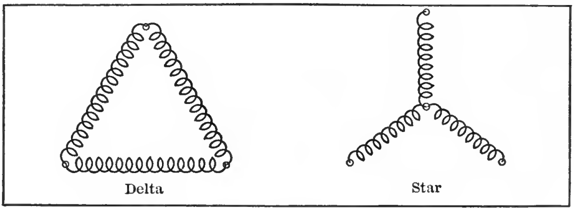


Fig. 6.—Star and Delta Coil Connections.

for each change of current, it is evident that it moves 30 electrical degrees for each change of 30 degrees in the current. Consequently, the flux revolves in synchronism with the current, or it moves twice the pole pitch for each complete cycle of current.

We could in the same way show that if the stator were wound for two phases instead of for three, and if two-phase currents were supplied to it, a rotating magnetic field would be set up as before. This the reader can readily verify for himself.

It will also be apparent that if the direction of the currents in two of the windings be reversed, the direction of rotation of the field, and consequently that of the rotor, will be reversed. This is done in the case of the actual motor by simply reversing the connections of two of the three leads. This is possible, since in practice the three windings of the three phases are always connected together in a Y or a delta connection, as shown in Fig. 6. Thus three wires only, instead of six, are required to supply current to the motor.

Production of Current in the Rotor

If in a stator such as that just described we place the revolving field of an alternator, the machine will in all essentials be an alternating-current generator and may be operated as such. Suppose, however, that we supply current to this stator so that a rotating magnetic flux is set up in the way just described, and let us drive the field by applying power through the shaft at exactly the same speed as that of the rotating magnetic flux due to the stator current. If now the driving force be removed, it is evident that the poles of the rotating field will be attracted by the magnetism set up by the stator current, and will be dragged around by them. The field will then continue to revolve at synchronous speed,

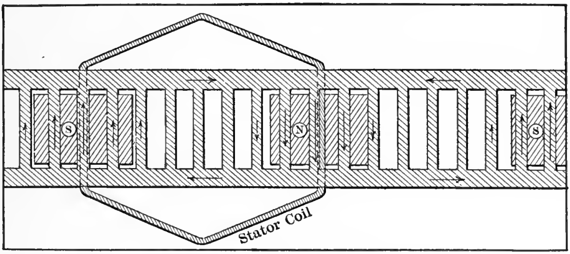


Fig. 7.—Distribution of Currents in Rotor of an Induction Motor.

and will maintain the same speed irrespective of the load applied, providing the torque is not so great as to cause the poles to pull entirely away. Such an arrangement constitutes a synchronous motor. The action in the case of an induction motor is similar, except that the current to magnetize the rotor, instead of being supplied in the shape of direct current from the outside, is produced in the rotor by e.m.f. generated therein. Moreover, the current in the rotor, instead of being direct, is a low-frequency alternating current.

Just how this current is produced in the rotor will be apparent from Fig. 7. This is intended to represent a portion of a squirrel-cage rotor winding. The observer is supposed to be looking out from the center of the rotor. The rectangles marked N and S represent the poles of the stator, passing from right to left over the stator and rotor. These may be thought of as the actual poles of a field magnet. As a matter of fact,

in the induction motor the poles do not exist in a physical sense, but the passing of the magnetism produces substantially the same effect as would the passage of actual poles. The principal difference is that in the induction motor the limits of the poles are not definite as shown, but the magnetism decreases gradually from a maximum to zero, and then increases in the opposite direction.

It will be seen from Fig. 7 that an alternating e.m.f. will be generated in the conductors of both the rotor and the stator by the moving flux. This e.m.f. will be strongest in those conductors at the middle of the poles, and will become less as they move from the middle, zero at the point midway between the poles, and a maximum in the opposite direction at the middle of the next pole. The magnitude of this e.m.f. is proportional to the flux cut, and to the relative speed of the conductor and the flux. It will therefore be greatest when the rotor is at rest, will decrease as the rotor speed increases, and will become zero at synchronism, i.e., when the rotor is moving at the same rate as the flux.

The frequency of the rotor e.m.f. is also variable. At rest it is evidently the same as the stator frequency. As the speed of the rotor increases, the cutting will be less rapid, since the rotor is now moving in the same direction as the magnetism. At the synchronous speed, the cutting is zero and consequently the frequency is zero also.

The currents set up by these e.m.f.'s must next be considered. If the rotor had only resistance but no reactance, the current and the e.m.f. would be in the same phase, and might be represented by the same arrows. Since, however, the rotor has reactance, the current will lag behind the e.m.f.

It will be seen at once that the currents in the rotor are in such a direction as to produce a torque in connection with the stator magnetism. Consequently, the rotor will start from rest. As the speed increases, however, the e.m.f. generated in the rotor will decrease, as was just pointed out, and will become zero at synchronism. Hence, the rotor will not quite reach this speed, but will attain only such a speed that the current will be just sufficient to develop the torque needed to maintain the rotation. If there is no load on the motor this speed will be very near synchronism, differing from it by only a small fraction of one per cent. When the load is heavy, however, the difference between the speed and the synchronous speed must be greater, and will amount to from 2 to 10 per cent in the majority of cases. This difference between the synchronous speed and the actual speed, divided by the former, is called the slip.

In the above explanation the rotor was assumed to be of the squirrel-cage type. It might, instead, have been assumed as of the wound rotor type. In this case it would have been provided with a three-phase winding similar to that upon the stator. The e.m.f.'s generated would have been the same, but the currents would have been somewhat different. In Fig. 7 the current is practically free to take any path it chooses. In the case of the wound-rotor machine, it is constrained to follow through the conductors in the order in which they are connected. Thus, in the squirrel-cage rotor the current differs from bar to bar. In the wound rotor there are a certain number of definite bands of current, the current being constant all over each of these bands. This fact is in itself somewhat disadvantageous, but the use of this type of winding presents certain advantages, as will be pointed out presently.

Shape of Flux Wave

In the foregoing, it was assumed that the stator current was the only current acting. This would be nearly the case if the rotor had no winding, or if it were of the wound-rotor type, and the rotor circuit were open. In the case of a motor in actual operation, neither of these suppositions is true. We must now consider the effect of the rotor currents in modifying the distribution of the flux which would be given by the stator current alone.

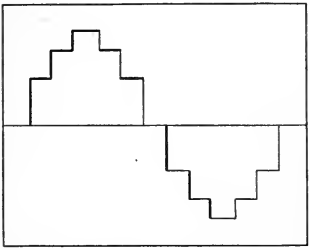


Fig. 8.—Magneto Motive Force Curve of Three-phase Induction Motor.

If we pass three direct currents of the value of the components of the three phases shown in Fig. 4, at the time 1, the magnetomotive forces acting across the gap will be represented by the broken line of Fig. 8. The curve of flux would be of the same general shape, but would have the corners somewhat rounded on account of leakage of the flux. On passing to the current at the point 2, the curve changes to the shape shown in Fig. 9, and becomes flatter on top, since there is no current in the phase B at this time. At the point 3 the shape is again that of Fig. 8, and so on. These considerations would then tend to show that the flux wave is continually changing from the shape of Fig. 8 to that of Fig. 9.

It will be noted that since a complete cycle of the change takes place in 60 degrees, its frequency is six times that of the frequency of the circuit. As a matter of fact, however, numerous tests show that when the applied wave of e.m.f. is a sine wave, the curve of flux is also of the sine shape and revolves with a uniform velocity. There are several reasons for this fact.

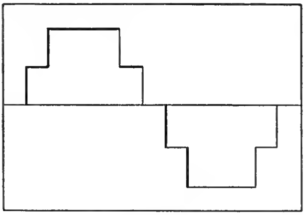


FIG. 9.—Magneto Motive Force Curves of Three-phase Induction Motor.

In the first place, the rate of cutting of the flux is determined by the shape of the wave of applied e.m.f. and not directly by the shape of the current wave. This is true on account of the fact that in any motor or transformer the counter e.m.f. is almost equal to the applied e.m.f. The difference is only enough to maintain the current, and this difference is always small compared to the applied e.m.f. Consequently if the applied e.m.f. is a sine wave, the counter e.m.f. is also practically of the sine shape.

A sine wave of rotating flux, revolving with uniform velocity, will generate a sine wave of e.m.f. in a single conductor. Consequently, it will also generate a sine wave of e.m.f. in *any* combination of conductors, since any number of sine waves of the same frequency added together at any phase relations will still be a sine wave. A sine wave of flux therefore corresponds to the most general case. It is not contended that the sine wave of flux is the only form of wave which can produce a sine wave of e.m.f. provided certain groupings of conductors were adopted, but it is evident that it is the only one which will always do so. Hence, it is the one which should be used in general calculations.

It may not be evident at first sight how a sine wave of flux can be generated with the currents as shown in Fig. 5. In the first place, the *currents* will *not* follow a sine shape. This is especially true at no load. The shape of the current wave is determined by the *difference* between the curves of applied and counter e.m.f. The difference of the two waves may depart quite markedly from the sine shape, without either of the two original waves being materially distorted, and consequently the current curve may be distorted without serious distortion of either of the two e.m.f. waves.

The influence of the rotor currents on the stator flux is also of great importance. Imagine the rotor operating at synchronism. There is

no cutting of the flux by the rotor conductors and consequently no current in the rotor so long as the shape of the flux wave does not change. It is evident, however, that the slightest change in the shape of the flux curve, or the slightest change in its rate of rotation will at once cause an e.m.f. to be generated, and since the resistance and the reactance of the rotor are both low, the resulting current will be large. According to the general law of induction, the direction of the secondary current will be such as to tend to stop the change of flux producing it. This effect, it will be noted, is particularly strong in the case of low-resistance squirrel-cage rotors, and less so in wound rotors and in cases where there is considerable resistance in the secondary circuits. A rotor having many bars will be more efficient in preventing fluctations than one having only a few. This is true, since any shifting of the flux within a tooth of the rotor may take place without setting up a corrective current, so long as the total flux through the tooth does not change. Obviously, the smaller the tooth, the less the possible fluctuation.

Vector Relations of Current and E.M.F.

We are now in a position to return to Fig. 7, and consider more in detail the relations of the current and the e.m.f. Let us assume first that the rotor is operating at synchronism. This would require the application of a small amount of power to the rotor shaft, to overcome the friction and windage. This condition is practically realized in the case of a motor operating without load, and having a low resistance secondary. The applied e.m.f. will be considered to be of the sine shape. This will always be assumed to be the case unless the contrary is stated. The distribution of the flux in the gap will therefore be sinusoidal, as just explained.

Since the rotor is operating at synchronism, there is no cutting of the flux by the rotor conductors, and consequently there is no rotor current except the small currents caused by the attempt of the flux to change its shape. These will have only a small effect, and will be neglected. The e.m.f. generated in the stator conductors by the flux may be represented by the arrows on the rotor bars. So long as the same flux cuts the stator and the rotor conductors, the e.m.f.'s generated in the two will be strictly in phase. This statement refers to the counter generated e.m.f., not to the applied e.m.f. It is evident from the figure that the generated e.m.f. is 90 degrees, or one-quarter period behind the flux. This is a general rule, and is always true in induction motors, alternators, transformers,

etc. Thus in the single-stator coil shown in Fig. 7, the e.m.f. is at the instant a maximum while the flux is zero. It is true that flux passes into the coil from both poles shown, but it also passes out again before surrounding either conductor. Hence, none passes *through* the coil, or we say the flux in the coil is zero. The generated e.m.f. therefore lags behind the flux by 90 degrees.

In speaking of the angle between the flux and the e.m.f., we make the convention that a flux and the current producing it are in the same phase when they reach their maxima at the same time. This is sometimes expressed as equality of time-phase, in distinction to space-phase. Thus, we consider a flux as in the same time-phase as the current producing it, although they are actually at right angles in space.

When the motor is operating as above at no load and at synchronous speed, the current in the stator is determined almost entirely by the magnetomotive force necessary to maintain the flux across the gap. This current can therefore be reduced by making the gap as short as possible. The magnitude of this current will in general be from 15 to 50 per cent of the full load current. To keep it within even these bounds it is necessary to use very short gaps. These will range from 0.02 in. or even less, to 0.125 in.

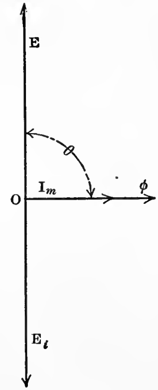


FIG. 10. — No-load Vector Diagram of Induction Motor.

In Fig. 10 is shown the vector diagram of the motor when operating at synchronism and under no load. Since the only current acting is that in the stator, the flux is in phase with the current. This is a general rule of all magnetic circuits. Where more than one current acts on a given magnetic circuit, the flux will be in phase with the resultant of all the currents acting. Thus in an induction motor under load, the flux will be in phase with and proportional to the resultant of both the stator and the rotor magnetomotive forces. It is true that there is a slight phase difference due to hysteresis, but this is small enough to be neglected for our present purposes. The student should thoroughly understand these two principles, that the induced e.m.f. lags 90 degrees behind the flux, and that the flux is in phase with the resultant of all the currents acting. All the vector diagrams are based on these two simple facts.

In Fig. 10 the horizontal line is taken as the axis of the flux, and since

the current is in phase with the flux, it is the current axis as well. The rotation of the vectors might be taken in either direction. For our purposes we will make the more common assumption of counter-clockwise rotation. The generated e.m.f. in both the stator and rotor will be represented by a vector 90 degrees behind the flux, or by OE_i. Since there is no cutting of the flux, the rotor e.m.f. is zero, but the line OE_i indicates its direction under all circumstances of load. The primary *applied* e.m.f. will be represented by a line equal and opposite to OE_i or OE. At a matter of fact, the applied e.m.f. will of course have to be somewhat larger than the counter e.m.f., and at such an angle to it as to maintain the required current. The difference, however, is not large, and for the present it will be taken as equal and opposite.

There is another small inaccuracy in the diagram as drawn. The power supplied as shown would be zero, since the current and e.m.f. are 90 degrees apart in phase, and the power is given by $EI \cos \theta$. This is not strictly the case, since a small amount of power must be supplied to make up for the losses in the iron and copper.

Now consider that full load is put on the motor. Under these circumstances it will not rotate at synchronous speed, but will drop behind enough so that sufficient e.m.f. will be generated in the rotor to maintain the current necessary to produce the torque. The flux will remain practically the same, since is it always just sufficient to generate a counter e.m.f. almost equal to the applied e.m.f. The difference between the two must, however, be greater now than when the motor was unloaded, since there is more current. The flux will therefore decrease about 3 per cent in a moderate-size motor.

The e.m.f.'s in the rotor and stator will have the same relation to the flux as in the case of no load. The principal difference is that we now have current in the rotor. The phase relation of this current to the e.m.f. of the rotor is of the greatest importance. In a circuit carrying an alternating current, the current will lag behind the applied e.m.f. whenever the circuit has inductive reactance as well as resistance. A circuit has inductive reactance whenever there are leakage lines of magnetic flux cutting the circuit and consequently generating in it an e.m.f. in addition to the applied e.m.f. This generated e.m.f. is always in such a direction as to tend to make the current lag. The rotor has such a leakage flux, since, on account of the current in the end rings and in the bars where they project beyond the slots, flux surrounds these parts. There is also some rotor leakage flux around the bars within the slots. All these leakage fluxes cause the current to lag.

It will be remembered, however, that the frequency of the current in the rotor is not constant, but varies from zero at synchronism to the line frequency at standstill. The magnitude of the e.m.f. generated, due to this leakage flux, will therefore be variable, being proportional to the rotor frequency and consequently to the slip. The lag of the rotor current behind the rotor e.m.f. will likewise be variable, being zero at synchronism and nearly 90 degrees at standstill.

As soon as we put load on a motor, current is produced in the rotor, and this current tends to produce a flux through the air gap. This flux would be in the same direction as the rotor current and consequently nearly at right angles to the main flux. It would therefore tend in general to increase the total flux. We have seen, however, that this flux cannot change materially, since the counter e.m.f. must always be approximately equal to the applied e.m.f. The total resultant m.m.f. of the entire magnetic circuit must therefore remain nearly constant, and must also be in phase with the flux. What happens, therefore, is that the flux decreases slightly, just enough to allow sufficient additional current to flow in the stator to offset the magnetizing action of the rotor current. For simplicity we may assume that the number of conductors in the rotor is the same as the number in the stator. In this case the *extra* current in the stator will be exactly opposite to that in the rotor. In any event, the added m.m.f. of the stator current will be equal and opposite to the rotor m.m.f.

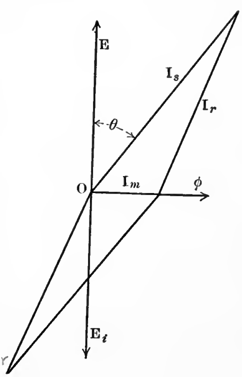


FIG. 11. — Full-load Vector Diagram of Induction Motor.

As we have shown, the primary applied e.m.f. is approximately 90 degrees ahead of the flux. The rotor current lags somewhat more than 90 degrees behind the flux. Since the added primary current is equal and opposite to the rotor current, it must lead the flux somewhat less than 90 degrees. The total stator current is the vector sum of the magnetizing current and the component which offsets the rotor current. Fig. 11 shows the relation of these quantities. As is always the case, the applied e.m.f. is approximately 90 degrees ahead of the flux. The rotor current is indicated by I_r, lagging somewhat behind the rotor e.m.f.

The primary current is represented by I_s, and is of such a value as to offset the rotor current, and at the same time supply the magnetizing current I_m.

Starting Conditions

When the rotor is at rest and power is applied to start it, the frequency in the rotor is the same as in the stator. Hence it is evident that the lag of the current behind the e.m.f. will be great. In fact, if the rotor resistance were zero, the lag would be 90 degrees. With an ordinary squirrel-cage rotor the lag will be about 70 degrees. The current is of course

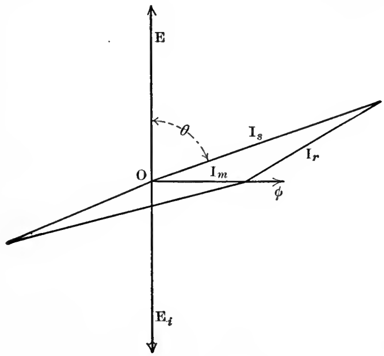


Fig. 12.—Vector Diagram of Induction Motor at Moment of Starting.

large, and since the stator must carry a correspondingly large component of current to offset this rotor current, the power-factor of the motor is very low. The vector diagram for this case is shown in Fig. 12.

The torque of any motor is proportional to the product of the flux and the rotor current. It is also influenced by the angle between the two, being a maximum when the angle is 90 degrees, and zero when it is zero degrees. This is readily seen in the case of a shunt-wound, direct-current motor. If the armature be so placed that the current passes first to those conductors situated midway between the poles, i.e., if the brushes are in the neutral position, the torque will be a maximum (this neglects the distorting action of the armature current). If, on the other hand, the brushes are placed 90 degrees from this position, the torque will be zero. The same thing is true of the induction motor.

Consequently the torque will be greatest for a given rotor current and a given flux when the rotor current is 90 degrees behind the flux or in phase with the e.m.f. generated in the rotor.

From the foregoing, it will be seen that the starting torque for a given current will be small in a motor so constructed that the lag of the rotor current is large. This is the case with the ordinary squirrel-cage motor. It is true that every effort is made to reduce the reactance of the rotor, but it is also true that the resistance of the rotor must be kept low in order that the copper loss and the slip may be low. In order to provide even moderate starting torque, it is, however, necessary to make the resistance of the rotor much higher than would otherwise be desirable. This is done by the use of brass end rings instead of copper, by using copper rings of small section, or by other suitable means. It is obvious that the best performance requires large rotor resistance during starting and low running resistance. This condition is frequently secured by the use of a rotor provided with a winding similar to the stator winding, the terminals being brought to three slip rings, and connected to external resistances or short-circuited as occasion may require. This constitutes what is known as a wound-rotor machine. It is inferior to the squirrel-cage motor in every respect, except that it has improved starting properties and that the speed is readily adjustable. The cost is of course higher. On account of these facts probably three out of every four motors sold are of the squirrel-cage type. This subject of starting torque will be treated more fully in a later chapter.

CHAPTER II

THEORY OF THE INDUCTION MOTOR

In the previous chapter we have considered somewhat the elementary theory of the induction motor. It is now necessary to take into account the effect of the magnetic leakage in the stator and the rotor. There are three fluxes that we must consider. These are the flux passing into both stator and rotor, the flux in the stator alone and the flux in the rotor alone. Of these three the first is the useful flux of the motor. The other two are the leakage fluxes and should be reduced to as small proportions as possible. The performance of the motor can perhaps best be understood by considering it as a transformer. It can readily be shown that if in the case of a wound-rotor machine, the rotor be held stationary and the resistance in the rotor circuit be varied, the performance of the machine in its general electrical respects is the same as though the rotor rings were short-circuited, the machine were in motion, and the load on the motor were varied. This fact enables us to treat the induction motor in all respects as though it were a transformer, and leads to a considerable simplification of the calculations. It also has the obvious advantage of developing the theory of the transformer without extra labor.

Consider a transformer supplying current to a non-inductive circuit, or an induction motor with its rotor at rest and its slip rings connected to non-inductive resistors. The resistors should be such as to give a balanced secondary load. Let the e.m.f. per rotor circuit be E, the rotor resistance R_r, the external resistance R, and the rotor inductance L_r. If then the frequency be f, the angular velocity of the rotating field will be $\omega = 2\pi f$. We then have for the rotor current per phase,

$$I = \frac{E}{\sqrt{(R+R_r)^2 + L_r^2\omega^2}}.$$

In the case of the motor with the rotor revolving in the usual way, we have a similar equation, but we have to take account of the fact that both the rotor e.m.f. and the rotor frequency are less than in the case of the

motor with stationary rotor. This is due to the fact that both the rotating magnetic field and the rotor are revolving in the same direction, and hence the rate of cutting is less. For example, if the rotor is running at 95 per cent of its synchronous speed, the rate of cutting will obviously be only 5 per cent as great as would be the case if the rotor were at rest. Likewise the frequency will be only 5 per cent of the primary frequency. The equation for the rotor current is then

$$I=\frac{sE}{\sqrt{R_r^2+s^2L_r^2\omega^2}},$$

where s is the percentage of slip. This can readily be changed to the form,

$$I=\frac{E}{\sqrt{\left(\frac{R_r}{s}\right)^2+L_r^2\omega^2}}.$$

The expressions in the case of the induction motor and in that of the transformer are the same, except for the term denoting the resistance. In the case of the transformer this term is (R_r+R), and in the case of the motor it is $\frac{R_r}{s}$. Hence it is evident that if instead of allowing a motor to rotate freely with a slip s and a rotor resistance R_r, we hold the rotor so it cannot revolve, and add enough external resistance so the total resistance is equal to $\frac{R_r}{s}$, all the electrical conditions will be the same as before, and we can treat the motor in all respects as a stationary transformer.

Another great simplification of our equations can be made by assuming a suitable number of turns on the rotor. For example, it will be evident at once that doubling the number of turns on the rotor and at the same time making the cross-section of the conductor half as great, would in no way change the electrical characteristics of the machine. Hence, in general we are at liberty, in developing the theory, to choose the ratio of the rotor to stator winding which gives the simplest relation. For this reason we choose the ratio of one to one.

An ideal, perfect transformer, i.e., one with no losses and with no magnetic leakage in either primary or secondary, has no effect on the circuit in which it is inserted, provided the ratio is one to one. The current and the e.m.f. in the secondary will be exactly equal to the primary current and e.m.f., and there will be no phase displacement between

the primary and secondary currents, since there will be no leakage flux, i.e., flux surrounding only one of the windings. This is also evident from the fact that since there are no losses, and since the currents and the e.m.fs. are equal, the power-factors on the two sides must be equal. Such a transformer could then be replaced by a wire of zero resistance and inductance.

If now we take sucn an ıdeal transformer and add to it the elements we have neglected, we shall have an apparatus which will act in all respects like the ordinary commercial transformer. The local resistance and reactance of the primary and secondary can be replaced by coils having the same reactance and resistance. The core loss we may replace by a non-inductive resistance consuming the same amount of power at the same voltage as the core loss, and we can use an inductance having no resistance to take a current equal to the magnetizing current. The equivalent circuit of such a transformer is shown in Fig. 13.

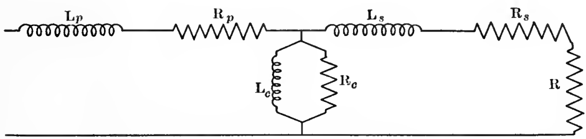


Fig. 13.—Equivalent Electric Circuits of Transformer or Induction Motor.

This diagram gives an exact equivalent of the circuit of a commercial transformer, with the following slight exceptions. It is not exact if the voltage varies, since in the equivalent circuit the magnetizing current passing through L_c is proportional to the voltage. In the commercial transformer this is not strictly true on account of saturation of the magnetic circuit. Likewise in the equivalent circuit, the loss corresponding to the core loss, i.e., the loss in R_c, is proportional to the square of the voltage. This is true of that part of the core loss due to eddy currents, but is not true of that part due to hysteresis. However, the discrepancy is slight, and, moreover, both transformers and induction motors are usually operated at constant potential. Their deviations from the exact facts, therefore, introduce no appreciable error.

The only important divergence of the theory from the facts is that in developing the equations we treat the inductance of the primary and secondary as though they were constant. This is not entirely true in the actual transformer, since on account of magnetic saturation, the

inductance becomes less as the current increases. However, a large part of the path of the leakage lines is through the air, and the change in inductance is slight and may well be neglected.

It is entirely possible to treat the circuit of Fig. 13 analytically, and the conclusions will be exact except in the slight particulars noted; but a further modification of the diagram is possible which, while not introducing any serious error, will greatly simplify our calculations. This change consists in considering that the magnetizing current and the core-loss current are taken directly from the line, instead of having to pass first through the resistance and inductance of the primary. The modified circuit is shown in Fig. 14. Since the magnetizing current and the core-loss current are small, the error introduced will be negligible.

We now proceed to prove that with this arrangement of the circuit, the ends of the current vectors of both the primary and the secondary

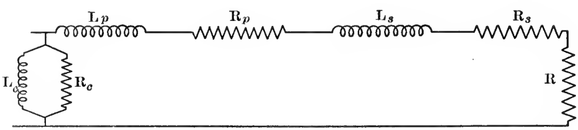


FIG. 14.—Modified Equivalent Circuit of a Transformer or Induction Motor.

currents, if plotted with their proper magnitude and phase position, will all fall on a circle. This generalization is of the greatest value to the student of the induction motor and transformer, as it not only enables him to scale all the values of the current, etc., directly from a simple diagram, but it also is of the greatest use in giving a remarkably clear mental picture of the relation of the various quantities involved. It is also of value in the testing of induction motors.

To prove this fact is comparatively simple. Using the same values as in Fig. 14, the current in the part of the circuit beyond L_c and R_c is given by the expression,

$$I=\frac{E}{\sqrt{\overline{R_p+R_s+R}^2+\overline{L_p+L_s}^2\omega^2}}.$$

This is the current in the secondary (rotor). The current in the primary (stator) is the same plus the currents taken by L_c and R_c. This current must of course be added vectorially. The secondary current, on account

of the inductance in the circuit, lags behind the e.m.f. The sine of the angle of lag is equal to the reactance divided by the impedance, or

$$\sin\theta=\frac{\omega\overline{L_p+L_s}}{\sqrt{\overline{R_p+R_s+R}^2+\overline{L_p+L_s}^2\omega^2}}.$$

Substituting this value in the expression for the current, we get

$$I=\frac{E\sin\theta}{\omega\overline{L_p+L_s}}.$$

This is the polar equation of a circle of diameter $\dfrac{E}{\omega\overline{L_p+L_s}}$. That this is so will readily appear from Fig. 15. Taking O as the origin, let

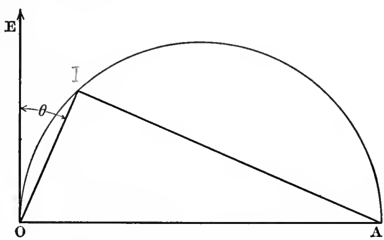


FIG. 15.—Vector Relations of Circuit with Constant Reactance and Variable Resistance.

the vertical line OE represent the applied e.m.f. The current will be represented by some such line as OI behind the e.m.f. by the angle θ. Through the points O and I draw a circle tangent to the line OE, and draw the line IA. The angle IAO is evidently equal to θ. If we assume the diameter of the circle OA to be equal to $\dfrac{E}{\omega\overline{L_p+L_s}}$, it is evident that the line OI will be equal to $\dfrac{E\sin\theta}{\omega(L_p+L_s)}$. Hence, the expression is the equation of a circle whose diameter is

$$\frac{E}{\omega\overline{L_p+L_s}}.$$

To obtain the primary current we must add to the current just found, the current through L and R. This current will lag behind the primary

e.m.f. by such an angle that $\cos\theta = \frac{R_c}{\sqrt{R_c^2 + L_c^2\omega^2}}$. In the actual motor this angle will be large, since in most cases the magnetizing current lagging 90 degrees will be considerably larger than the power component of the current to supply the losses. In Fig. 16, the line OB is drawn to represent this current. The total primary current then is the vector sum of BO and OI or BI. To complicate the diagram less, the vector representing the e.m.f. is drawn from the point B.

Several things are at once apparent from the diagram. If we take any value of the primary current as BI, we can separate it into two components ID and IC, the first parallel to the e.m.f. and the second perpendicular to it. The former is the power component of the primary current, the latter is the wattless component. By using the proper scale,

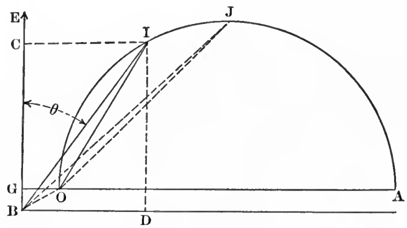


FIG. 16.—Circle Diagram of the Induction Motor.

it is evident that the power component of the current can be considered as the power itself. This is true only when, as is usually the case, the e.m.f. is constant. Thus at the current I, the power input is BC, and of this, the portion BG is wasted in the core loss. The maximum power input into the primary is evidently attained when the current is represented by BJ. The power-factor of the secondary is then 70 per cent, or the angle of lag of the secondary current is 45 degrees. It will readily be seen that this is equivalent to saying that the reactance and resistance of the secondary must be equal for maximum input. The maximum value of the power-factor is obtained at such a value of the current that the current vector is tangent to the circle.

We have previously shown that the construction holds equally well in the case of the induction motor. Its application to the testing of an actual motor is shown in Fig. 17. The motor had a rotor of the squirrel-cage variety.

TABLE I

NO-LOAD READINGS

Volts.	Amp. per phase.	Watts.	Power factor $W \div (\sqrt{3}EI$	Power component of current.	Wattles component of current.
440	10.5	3805	0.476	5.00	9.23

LOCKED-ROTOR READINGS

Volts.	Amp.	Watts.	Corrected values: Volts.	Corrected values: Amp.	Corrected values: Watts.	Power factor	Power component of current	Wattless component of current
220	41.9	4290	440	83.8	17160	0.269	22.5	80.8

Res. per stator phase (Y wound) $=0.397$ ohm.

Stator loss at start $=3\times0.397\times83.84=8400$ watts.

Rotor loss at start $=17160-8400-3805=4955$ watts.

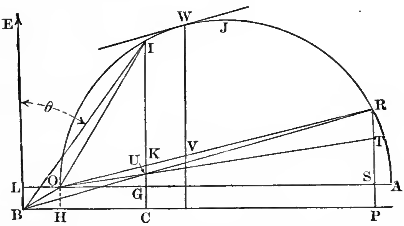


FIG. 17.—Circle Diagram of the Induction Motor.

The data used in constructing Fig. 17 are given in Table 1. In taking these data, only an ammeter, voltmeter, and wattmeter were used. Only two readings were taken, one with the rotor running freely without load, and the other with the rotor locked, so it could not rotate. In taking this latter reading, it is usually impracticable to apply the full voltage to the motor, since it would heat up very rapidly under these circumstances. The readings were obtained by applying half of normal voltage and multiplying the current observed by the ratio of the rated voltage to the observed voltage, and the computed watts by multiplying the observed watts by the square of the same ratio. The power-factor is obtained in the usual way by dividing the real power by the apparent power; or,

$$\cos\theta=\frac{P}{\sqrt{3}EI}.$$

Taking then the watts consumed at no load, the value of the power component of the current is given by the expression $I \cos \theta$, and the value of the wattless component is similarly given by $I \sin \theta$. These two components are plotted respectively as OH and OL. Similarly, the two components of the starting current are determined and plotted as PR and a horizontal line (not drawn) from R to EB. This line is equal to BP. Since the flux is approximately constant the core loss will also be constant, and will be equal to the distance between the lines LA and BP.

Since at the point R the rotor is stationary, the output is zero and all the power put into the motor is wasted. The line PR represents the power component of the current, or to a proper scale the power itself. Hence it represents the total loss in the motor at standstill. Of this loss the portion PS is the stator iron loss. The remainder must be copper loss in the stator and rotor and the rotor iron loss. This copper loss consists of two parts, the loss in the stator and that in the rotor. Let us therefore divide it into two parts by a point T. RT is then the rotor copper loss and ST is the stator copper loss, or more exactly the copper loss in the stator due to the component of the stator current which offsets the rotor current. The copper loss due to the magnetizing current is already included in the loss PS.

If we join the points R and T to the point O by straight lines we can readily determine the losses at any load. Thus let the primary current be BI, and draw the vertical line IC. Then CG is the core loss and the copper loss due to the no-load current BO, GU is the stator loss and UK is the rotor copper loss. The total input into the machine is IC. Taking out the losses just mentioned, the remainder IK is the output of the machine. The efficiency is obviously IK divided by IC.

That the total copper loss with the exception of that due to the component representing the no-load current is represented by the intercept GK may be readily shown as follows. What we have to prove is that the intercept SR and GK have the same ratio as the squares of the lines representing the respective currents, or,

$$\frac{GK}{SR}=\left(\frac{OI}{OR}\right)^2.$$

But

$$\frac{GK}{SR}=\frac{OG}{OS}=\frac{OI\cos(AOI)}{OR\cos(AOR)}=\frac{OI\cdot\frac{OI}{OA}}{OR\cdot\frac{OR}{OA}}=\left(\frac{OI}{OR}\right)^2,$$

therefore the proposition is proved.

A number of other quantities can be at once taken from the diagram. The primary power-factor is IC divided by IB, or it is cos θ. The maximum power output of the motor is WV, in which W is defined by the point of tangency of a line drawn parallel to OR. This is so, since this is the longest possible line for the output. The maximum input is at the point J. (See Fig. 16.) It might seem that the points of maximum input and output should be the same. They are nearly the same, but differ somewhat on account of the change in efficiency, with the load.

Slip and Torque

Before showing how we can obtain the values for the slip and the torque from the diagram, it is necessary to prove two propositions. The first of these is as follows: The slip expressed in percentage of the full-load speed is equal to the rotor copper loss divided by the input to the rotor. As has been previously shown, the rotor current is given by

$$I=\frac{sE}{\sqrt{R^2+s^2L^2\omega^2}}.$$

The rotor copper loss is

$$I^2R=\frac{s^2E^2R}{R^2+L^2\omega^2s^2}.$$

The total input into the rotor is

$$P=EI\cos\theta=\frac{EsE}{\sqrt{R^2+L^2s^2\omega^2}}\cdot\frac{R}{\sqrt{R^2+L^2s^2\omega^2}}=\frac{sE^2R}{R^2+s^2L^2\omega^2}.$$

The percentage of rotor loss is evidently the ratio of these two, or S. Hence the proposition is proved.

Torque is frequently defined in synchronous watts. By this we mean the watts that would be developed at the shaft if the motor were running at synchronous speed, and exerting the given torque. It is obvious that, if more convenient, we may express torque in synchronous kilowatts or in synchronous horse power. This latter is a very convenient way of expressing the torque. Thus if we say a certain 25-h.p. motor develops a starting torque of 50 synchonous h.p., we know at once, without calculation, the starting ability of the motor; that is, the motor will start a load requiring approximately double full-load torque. Moreover the torque is expressed without reference to the speed.

The mechanical output of the motor is evidently $D\,(1-S)$ where D is the synchronous torque. Since the percentage of rotor loss is as we have

just shown, equal to S, the slip, the output is also given by Input X $(1-S)$ Writing these two expressions equal to one another, we see at once that Input $=D$. Hence we may state the general rule, the synchronous torque is equal to the total rotor input; or $D=\frac{SE^2R}{R^2+S^2L^2\omega^2}$. This is true whether the rotor is at rest or in motion. It must be kept in mind, however, that both of these expressions have been derived on the supposition that we have sine waves of current and flux. If these conditions are not fulfilled, the torque will be less than indicated by the above. In any practical case we have also to make some deduction for the torque required to overcome friction. This is in general a small correction. On the other hand, the starting torque is increased very materially in many cases by the iron losses in the rotor. This is explained more in detail elsewhere. In consequence of these facts, the writer does not consider the method of determining the starting torque by computing the rotor lost a very practical one, but would prefer whenever possible to measure the torque directly.

From the two above propositions, we see at once from the diagram that the percentage of slip is given by UK divided by IU, and the torque is given by IU. In particular the starting torque is equal to TR. A motor designed to give the greatest possible starting torque should take a starting current just a trifle less than BJ, Fig. 16.

For convenience of the reader we add a table of the various quantities that can be scaled from the diagram of Fig. 17.

$BI=$ primary current;
$OI=$ secondary current;
$CI=$ input into motor;
$CG=$ fixed loss, i.e., hysteresis, eddy currents, and friction;
$GU=$ primary copper loss;
$KU=$ secondary copper loss;
$KI=$ output of motor;
$KI \div IC=$ efficiency of motor;
$CI \div BI=$ power factor;
$BO=$ idle current;
$KU \div UI=$ slip;
$UI=$ synchronous torque;
$TR=$ starting torque in synchronous watts;
$WV=$ maximum output of motor.

Referring to the simplified circle diagram of Fig. 18, which is obtained

from the form just given by assuming that the no load losses of the motor may be neglected, the ratio of the magnetizing current to the rotor current of the motor with the rotor locked and assuming that the rotor has only reactance and no resistance, is of great importance. This value is called the leakage coefficient, and is usually designated by the letter σ, or $\sigma=\frac{OA}{AB}$. The determination of this factor is treated in Chapter (8).

We can readily derive a relation between this factor and the maximum value of the power-factor of the motor. At the condition of maximum power-factor, the current vector is represented by a line such as OC drawn from O and tangent to the circle. The angle of lag θ is the angle

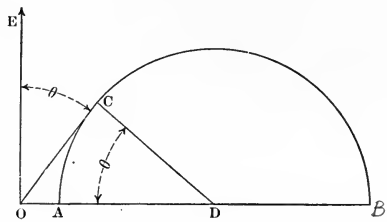


FIG. 18.

EOC, and this angle is equal to the angle CDO. We then have

$$\operatorname{Cos}\theta=\frac{CD}{OD}=\frac{AB}{2OD}=\frac{AB}{AB+2OA}=\frac{1}{1+2\frac{OA}{AB}};$$

and since

$$\sigma=\frac{OA}{AB},$$

maximum

$$\cos\theta=\frac{1}{1+2\sigma}.$$

Fig. 19 will serve to illustrate the difficulties encountered in attempting to construct a more exact diagram based on the connection shown in Fig. 13. We will start with the vector E_s representing the total voltage generated in the rotor of an induction motor, or the secondary of a transformer. Of this, a portion X_sI_s is used up in overcoming the reactance of the secondary circuit, and a portion R_sI_s in overcoming the resistance. In the transformer, X_s may be entirely due to the windings, or in the more general case, it may be partially due to the windings and partially

due to the reactance of the connected load. It may even readily happen that X_s is negative, if the transformer is supplying over-excited sychronous machinery, or a long transmission line, taking a leading current. In the induction motor, however, X_s is in general entirely due to the reactance of the windings alone. No object would be gained, and the characteristics of the motor would be seriously impaired by the use of additional reactance in this circuit. On the other hand, the use of negative reactance consisting of condensers would result in an improvement of the

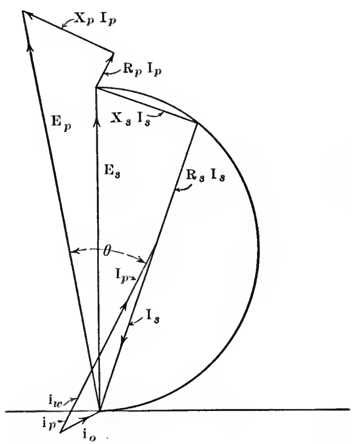


FIG. 19.—Exact Construction for Relation of Currents and e.m.fs. of an Induction Motor.

characteristics of the motor. It is, however, impractical to make use of this fact, since the condensers required at the low voltage and frequency of the rotor would be of prohibitive size.

Knowing then the value of the rotor reactance, and the current in the secondary, we can at once determine the length of the line X_sI_s. Its direction will be determined by the fact that its end must lie on the circle described upon E_s as a diameter. This will be apparent, since X_sI_s and R_sI_s are at right angles. The phase of the secondary current is the same as that of the vector R_sI_s.

The primary current will be equal to the secondary current, plus

the no-load current; the addition is of course to be made vectorially. The no-load current is dependent both in phase and magnitude upon the secondary or rotor e.m.f. E_s. The no-load current, taken in this way, is not quite the same as what we ordinarily call by this name, since it is necessarily measured before it enters the primary or stator, and consequently contains a power component due to the stator copper loss. The difference is, however, negligible.

If, then, for any given secondary e.m.f. we know the power lost in hysteresis and eddy currents, and the necessary magnetizing current, we can determine the components of the no-load current and lay it off in proper position and magnitude as shown in the diagram. The primary or stator current I_p is then given by the sum of I_s and i_0. It is now simple to find the primary applied e.m.f. by adding to the secondary e.m.f. E_s the vectors X_pI_p and R_pI_p, representing respectively the reactance drop and the resistance drop.

The difficulty of constructing this diagram will be apparent when we consider that we have to start with the voltage applied to the secondary, and this is not known until the diagram is completed, since the applied voltage, E_p, is constant, while the secondary e.m.f., E_s, continually decreases as the current is increased. It might seem that we could construct the diagram as shown, determine the ratio of E_p and E_s and their difference in phase, and then reconstruct the diagram, drawing E_s to the proper scale to start with. This would be correct, were it not for the fact that the magnetizing current i_0 is not proportional to the secondary voltage. Its power component increases about in proportion to the 1.8 power of the voltage, and its wattless component somewhat faster than the first power of the voltage, the rate of increase being greater as the saturation of the iron increases. We are therefore forced to the conclusion that the best method of procedure would be to adopt a method of trial and error, each new attempt giving us data so that the next attempt would be more nearly correct.

One advantage of the diagram of Fig. 19 is that if desirable we may include in the primary resistance and reactance the resistance and reactance of the lines supplying the motor. Thus we might investigate the action of the motor when supplied through a transmission line, the voltage at the beginning of the line being kept constant.

That the simpler diagram of Fig. 17 is sufficiently accurate and that it is in general undesirable to use the more complicated ones, particularly in the case of the induction motor, will be more apparent when we consider that all of these diagrams are based upon the waves of e.m.f. and

flux being sinusoidal. In the actual motor, with an applied sine wave of e.m.f., none of the other waves are strictly sinusoidal. With the motor well loaded, the stator current is nearly sinusoidal, but the no-load current, and still more, the rotor current are badly distorted. It will be seen that on this account the theory is by no means perfect, and an attempt to introduce great refinement in our diagrams is inadvisable.

CHAPTER III

STARTING TORQUE

It has been shown in a previous chapter that the starting torque of an induction motor may be expressed in synchronous watts, and it has been shown that the starting torque in synchronous watts is equal to the loss in the rotor. It is the intention in the present chapter to give a simple proof of this statement, and also to show that the same rule applies to the starting torque of any motor in which the field retains a definite form with respect to the rotor. The demonstration also applies to the case of the single-phase commutator type motor, in fact to any of the commercial types of motor with the exception of the single-phase induction motor.

To apply this rule we must understand by the " synchronous " speed of a machine, the speed which it would attain if it were allowed to run freely without friction and with the same field flux as that used in starting. By the " synchronous watts " we mean the power the machine would deliver if it were operated at the synchronous speed and exerted the given starting torque. It might be considered as the torque corresponding to a given output in watts at the synchronous speed. In a direct-current shunt motor at the instant of starting, let us consider that full-load current is in the rotor. The machine will exert full-load torque, since the torque in this case is obviously independent of the speed. The power expended in the armature and the resistance in series with it, whether it is at rest or in motion, is equal to EI. If the machine operates at full load and 100 per cent armature efficiency (which would be the case if it ran at the synchronous speed), the output in watts is the same as the input or it is EI. Hence the starting torque in synchronous watts is equal to the power expended in the armature circuit. This same rule also obviously applies irrespective of the speed of the rotor, or in general the torque exerted by the motor in synchronous watts is equal to the power expended in the rotor circuit.

To obtain the useful torque at the shaft it is necessary to decrease the torque as obtained above by the torque required to overcome the

mechanical friction of the motor, and that required to overcome the torque due to hysteresis. In general these corrections are of small magnitude.

In the case of the series motor, it is evident that the same argument will apply. We have here, however, a somewhat different case, since the syn hronous speed, as above defined, may be either more or less than the full-load speed. If less than full-load current is passed through the motor at starting, the field will be weak, and the "synchronous speed" as above defined will be greater than the full-load speed. If the starting current be greater than the full-load current, the "synchronous speed" will be less than full-load speed. The starting torque is represented by exactly the same expression as before. The series motor has therefore no advantage over the shunt motor in regard to starting torque, provided the starting current is the same as the running current. This is contrary to the general impression. The misconception arises from neglecting the consideration of the final speed arrived at by the motor. To consider a specific case, suppose a load offering a constant torque of 500 ft-lbs. is to be started and accelerated to 1050 rev. per min. Suppose further that this torque at this speed represents the full-load of the motor, or in this case 100 h.p. If current is supplied at 250 volts, the shunt motor will require about 332 amperes to start the load. The series motor will require slightly less, say 325 amperes, since there is no shunt field to be supplied. The one motor has then practically no advantage over the other.

Suppose the torque to be increased to 750 lbs. The shunt motor will require approximately 50 per cent more current to start, say 498 amperes. Approximately the same speed as before will be attained, and the output will be nearly 150 h.p. The series motor, on the other hand, will require only about 400 amperes to start, and attains a speed of 852 rev. per min. This is on the supposition that the fields are unsaturated. The h.p. output will consequently be 122 h.p. The input of 400 amperes at 250 volts is, allowing 10 per cent for losses, just equal to 122 h.p.

The case of the series single-phase motor is readily seen to be the same as that of the series direct-current motor.

The starting torque of the induction motor is of the greatest interest to us here. The facts in this case will perhaps be most readily seen if we consider the rotor of an induction motor to be at rest, and that a field magnet, excited by direct current, is rotated around it. The conditions will be the same as those in an induction motor during the starting period, providing the rotating magnetic field in the motor is constant in

strength, rotates at a uniform velocity, and preserves a uniform distribution. In the case of the rotating field, a certain torque will be required to maintain the motion. Neglecting the friction losses of the field, it is evident that the same torque will be exerted upon the rotor as upon the field. All the power applied with the exception of the small friction loss must be dissipated in the rotor. It is evident at once that the torque in synchronous watts is equal to the loss in the rotor. The case is the same in an induction motor, provided the conditions just mentioned are complied with. This is nearly the case in practice.

The above obviously applied both to the squirrel-cage and to the wound-rotor type of machine. It would seem then, that the starting would be equally efficient for either one. The difficulty with the squirrel-cage type arises, not because more power must be supplied to the rotor, but on account of the difficulty of getting it there. In the first place the copper loss in the rotor in normal operation, in a typical modern squirrel-cage motor, will be perhaps twice that in the stator. Hence in addition to the rotor loss, we have a loss of about half as much in the stator. This corresponds to the field loss in a shunt motor, but whereas the field loss in the shunt machine is very small, here it is a large proportion of the rotor loss. In fact in squirrel-cage motors built for small slip and consequently small starting torque, the stator loss may be as much as twice the rotor loss. This is avoided in the case of the wound-rotor machine, by causing the loss in the rotor to be temporarily several times as largeas the stator loss.

The other difficulty is that on account of the inductance of the induction-motor windings, the current is supplied at a very low power-factor. This means that a large current must be taken from the line, and that on account of the low power-factor, a great disturbance of voltage is produced. We may put the matter this way. Neglecting the field losses, the series motor, either for direct or alternating current, the shunt motor, and the induction motor all require the same amount of power in starting. This power is the same in watts as the starting torque in synchronous watts. In the series motor, the field loss is zero (since the resistance of the field takes the place of resistance which would otherwise have to be supplied). In the shunt motor the field loss is small, being from 3 per cent to 1 per cent or less. In the case of the induction motor, this loss is also small if a wound-rotor machine is used, and is about the same as in a shunt motor of corresponding size. In the squirrel-cage motor, however, this field loss is considerable, varying from 30 per cent to 300 per cent or more of the rotor loss.

The above is on the supposition that the shunt and series motors are supplied with current at a constant voltage, and that the induction motor is supplied with current at just the voltage to force the required current through the windings. This is done at least approximately in practice in the case of squirrel-cage induction motors by using an auto-transformer to give the correct voltage. If this could be done in the case of the shunt and series direct-current motors, their starting efficiency would of course greatly exceed that of the induction motor. Likewise in the induction motor with wound rotor, if the current produced in the secondary could be utilized, the starting efficiency would be very greatly increased. To do this, however, would be very difficult, since during the process of starting, the power would be supplied at both varying voltage and varying frequency. No practicable method of doing this is available at the present time unless we regard the case of two motors operating in cascade as being an exception. The reader should of course bear in mind that the production of torque alone requires theoretically no power. In practice, the only power required, provided advantage is taken of the methods above pointed out, is that required to supply the copper losses and in the case of the induction motor, the iron loss.

Starting Torque of Commercial Squirrel-cage Induction Motors

The question of the design of the squirrel-cage induction motor so as to give suitable starting torque is a very serious one. The designer has to choose between high starting torque and high efficiency. A slightly different way of expressing the starting torque may help to make this clearer. As was pointed out, the starting torque in synchronous watts is equal to the loss in the rotor. Suppose we have full-load current through the stator at starting. The rotor current will be approximately the same as the rotor current at full load, and consequently the rotor loss will be equal to the full-load rotor loss. The slip is equal to the rotor loss divided by the input to the rotor. Likewise the starting torque in percentage of the full-load starting torque is equal to the same quantity. Consequently the starting torque with full-load current is equal to the slip. For example, a motor with a slip of 4 per cent will have a starting torque of only 4 per cent when the motor takes full-load current. The torque increases, however, in proportion to the square of the current. An average motor will take about six times full-load current if thrown directly across the line. Consequently the motor considered would

develop a maximum starting torque of $6^2 \times 4 = 144$ per cent. This neglects the effect of the iron losses in increasing the torque. This increase will be treated later. The expression for the percentage of starting torque may then be written thus: $D = \left(\frac{I_c}{I_1}\right)^2 s$, where I_c is the current per phase taken when the motor is thrown directly across the line, I_1 is the full-load current, and s is the slip. Thus to increase the starting torque we can do either one of two things: increase the slip, or increase the starting current. To do the first is obviously undesirable, since it decreases the efficiency. It is likewise undesirable to increase the starting current beyond a certain value. The most obvious objection is that to do so impairs the regulation of the line, demands larger auto-transformers or rheostats for starting, and is apt to cause excessive heating of the windings of both stator and rotor.

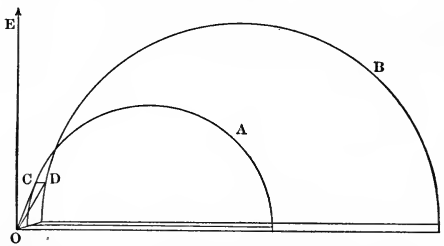


Fig. 20.—Comparative Circle Diagrams of Induction Motor with Few and Many Turns per Coil.

Of even more importance is the fact that increasing the starting current to too great a value makes the characteristics of the motor very poor. This is particularly true of the power-factor. This will be readily seen from Fig. 20. A represents the circle diagram of a motor in which the starting current is five times the full-load running current. The idle current is taken in both cases equal to one-fourth the full-load current. In curve B the starting current is taken as eight times the full-load current. C and D represents respectively the points on the two circles corresponding to full load. Since the cosines of the angles COE and DOE represent respectively the power-factors in the two cases, it is easily seen that the power-factor of the motor A, at full load, is much better than that of B. In fact the power-factor of A is better for all loads from zero to 175

per cent of full load. If the slip in both cases is 4 per cent the motor *A* will develop $5^2 \times 4 = 100$ per cent and the motor *B* $8^2 \times 4 = 256$ per cent starting torque. However, the motor *B* is so much inferior in power-factor throughout all the operating range, as to make it highly undesirable. Moreover, if the load to be started requires a greater starting torque than 100 per cent, we should get equally as good results by using motor *A* and *stepping up* the voltage for starting. For any given torque, say 150 per cent, the current required, and the power-factor would be the same in the two cases.

Before leaving this subject, it should be pointed out that the employment of a large starting current to secure large starting torque is, in general, more justified in the case of 25-cycle motors than in the case of 60-cycle motors. This is due to the fact that on account of the smaller number of poles, the power-factor in 25-cycle motors is inherently higher than in the case of the higher frequency; hence, the lowered power-factor is not so objectionable.

If, on the other hand, we attempt to secure large starting torque by increasing the slip, we reduce greatly the efficiency, and impair the speed regulation of the motor. Just what this means in dollars and cents may be shown by the following typical case. Consider a 100-h.p. motor for which it is specified that the starting torque must be at least 175 per cent with a current not exceeding 5.5 times full-load current. This is by no means an unusual requirement. The slip will be $175 \div 5.5^2 = 5.72$ per cent. When the motor is operating at full load, this means a constant rotor loss of approximately 6 h.p. or $4\frac{1}{2}$ k.w. If the motor is operated six hours per day, 300 days per year, and energy is sold at the low rate of 2 cents per k.w.-hr., the cost of the energy lost in the rotor yearly is $162. At least half of this could be saved by constructing the machine with a wound rotor of low resistance. This saving would justify an investment of $547, taking interest and depreciation at the high value of 15 per cent. This would be ample to cover the extra cost of a wound-rotor machine instead of the squirrel cage, or of a loose pulley and friction clutch on the counter shaft. This is a point which is frequently not given the consideration which its importance demands. A little effort to get easy starting conditions, and the use of a motor with small slip, will pay large dividends upon the time and money invested.

Comparative Starting Torques of Squirrel-cage and Wound-rotor Motors

It is a general impression that the induction motor with wound rotor is far superior in the matter of starting torque to the squirrel-cage machine. This is true to a certain extent, but to a far less degree than is generally supposed.

To take a typical case assume a 20-h.p., 60-cycle, 1200-rev. per. min. three-phase machine. This in the squirrel-cage type would develop a starting torque of about 200 per cent, taking a current of $5\frac{1}{2}$ times full-load current to do so. To start under full-load torque it would require 70.7 per cent of full voltage to be applied to it. The motor current would then be 70.7 per cent of 5.5 or 3.89 times full-load current. If we assume an auto-transformer to be used and allow nothing for the loss in it, the line current will be one-half of 5.5 or 2.75 times full-load current. To develop this torque will require an expenditure of 20 h.p. in the rotor. If the full-load copper loss in the stator is 4 per cent we shall have during starting a loss of $4 \times 3.89^2 = 60$ per cent, and if the iron loss at full voltage is 5 per cent, the iron loss during starting will be about 3 per cent. This is true since the iron loss decreases somewhat faster than the voltage. The total input will then be about 163 per cent of 20 h.p., or 32.6 h.p. To this something should be added on account of the loss in the auto-starter.

In the case of the wound-rotor machine, the starting current for 100 per cent torque would be approximately equal to the full-load current. The full line voltage would of course be applied. As before, we should have to expend 20 h.p. in the rotor. In addition we should have a loss of 4 per cent in the stator copper and 5 per cent in the stator iron loss. The total input is then 109 per cent of 20 h.p. or 21.8 h.p.

Comparing the two, it will be seen that the wound-rotor machine is somewhat superior in that it takes only 21.8 h.p. compared with 32.6 for its rival. The cost of the energy used in starting is therefore somewhat less. In the matter of current required it is decidedly superior, since it requires only one, compared with $2\frac{3}{4}$ times full-load current. Moreover the power-factor in the case of the wound-rotor machine is about 85 per cent compared with 35 per cent in the case of the other. If the motor is operated upon the circuits of a public service company, this is no great disadvantage to the user of the motor. It is, however, a decided one from the standpoint of the company, on account of the deleterious effect of low power-factor upon the regulation of the circuit. If the motor is of

a size comparable with the generator supplying the energy, it may be a very serious matter.

For example, consider the case of a 100-h.p. motor supplied from a 100 k.w. generator. Such a motor would require 120 amperes when operating under full load on a 440-volt, three-phase circuit. During starting, however, it would require, as we have shown, about $2\frac{3}{4}$ times as much current or 330 amperes. This is on the assumption that the motor is developing 100 per cent torque. If the generator operates at 460 volts to allow for drop in line, its full-load current will be 109 amperes. The generator has then during the starting of the motor a power load of 133 h.p., plus the loss in transmission line, or say 104 k.w., and an apparent load of 263 k.v.-amp. The power-factor of the load is 39.5 per cent. It is unlikely that under this large load the generator would be able to keep up its voltage, and it would probably be impossible to start the motor. Such a combination would work only in case the motor could be started with a very light load. If, on the other hand, the motor is of the wound-rotor type, the power load will be 81 k.w. and the apparent load will be 92 k.v.-amp. The power-factor will consequently be 88 per cent, and no difficulty should be experienced in starting under full load.

Influence of Poor Leakage Coefficient, in Wound-rotor Machines

To one who has studied most of the text-books upon induction motors, it is rather a surprise to find that in many cases a squirrel-cage motor is guaranteed to develop the same maximum torque as the corresponding wound-rotor machine of the same maker. This fact is partially explained by the fact that the torque is more irregular in a motor of the latter type. A factor of more importance is, however, the fact that the wound-rotor machine has a much larger leakage coefficient than the squirrel-cage machine. This is due to the longer end connections in the former, and to the fact that the currents are confined to definite paths instead of being allowed to pick their own path.

On account of the advisability of using standard windings, the same stator is usually used for a machine of a given rating, irrespective of the type of rotor. As a wound-rotor machine, the leakage coefficient will be approximately one-third greater than in the case of the squirrel-cage machine. Hence the circle diagram of the latter machine will be 33 per cent larger in diameter, and the current in the rotor at start 33 per cent greater. Since the starting torque is proportional to the square of the

current, the torque with the same equivalent rotor resistance would be 1.78 times as great. Of course the rotor resistance of the wound-rotor machine would in practice be made much more than that of the squirrel-cage machine, and this would usually be more than enough to counteract the better leakage coefficient. However, the other facts mentioned, such as lack of uniformity of the torque, would act to cut down this advantage, and, as previously stated, the torque of the squirrel-cage machine will in many cases be found to be fully as high as that of its rival. It should also be noted that on account of the lower leakage coefficient, the power-factor and the pull-out point of the squirrel-cage machine will be materially higher than in the case of the wound-rotor machine. Rewinding the machine with a smaller number of turns will, it is true, remedy the latter fault, but it leads to a large iron loss, and the power-factor will generally be somewhat poorer. On account of its inherently better characteristics, the squirrel-cage machine should be used wherever possible: The rotor should, however, be preferably of low resistance, so as not to offset the better power-factor and pull-out point by a lowered efficiency.

Effect of Iron Losses

We have seen that in order to develop any given torque, it is necessary that there be a loss in the rotor equal to the power that would be developed by the motor if running at synchronous speed with the given torque. It will be at once evident that it does not matter whether this loss is copper loss or iron loss. As a general thing, the iron loss is not of much assistance in increasing the starting torque, but in some cases it may make a decided difference. The writer has seen a number of cases in which the starting torque was increased at least 50 per cent by the iron loss.

To increase the torque in this way is decidedly advantageous, since this rotor iron loss disappears almost entirely, when the motor is in normal operation. This is due to the fact that the frequency in the rotor is very low when the motor is operating near synchronous speed. Consequently the rotor iron loss is available when wanted for starting, and disappears when the motor is up to speed and it is no longer needed. The iron loss mentioned is that normally due to the pulsation of the flux in the rotor, and should be carefully distinguished from the excessive loss sometimes found in the rotor teeth at or near synchronism. This phenomenon is treated in detail elsewhere. The effect of this if present is decidedly disadvantageous, since it is greatest near synchronism, and disappears at zero speed.

Fluctuation of Torque in Wound-rotor Machines

The case of the wound-rotor machine is somewhat less favorable than as above presented, since we have neglected the fact that the torque of a machine of this type is by no means constant, but varies widely in different positions of the rotor with respect to the stator. This fluctuation is present to some extent in the squirrel-cage motor, but is usually of negligible importance. It is greater, the greater the current and the fewer the number of phases in the rotor. This is largely the reason for the superiority of the squirrel-cage machine, since here the number of phases is very great.

If a wound-rotor machine be started under a small torque, say 50 per cent the fluctuation may amount to perhaps 25 per cent of the average torque. If full-load torque is developed, the fluctuation may be as much as 70 per cent. If we go to the limit and short-circuit the rotor, it will be found that the fluctuation is so great that the torque frequently drops to zero in certain positions, or even that the rotor locks in these places, and torque must be applied to get the rotor past the point, or the current must be shut off and a fresh start made. The rotor is of course not intended to be short-circircuited during the starting period, but it may easily happen when starting a heavy load with an automatic starter.

Various expedients are employed to reduce this fluctuation. Some of these are the use of short-pitch windings on the stator and rotor, making the rotor winding unsymmetrical, and the use of spiral slots in the rotor, produced by placing each rotor lamination at a slight angle to the one before it. By the use of one or more of these devices, it is possible to reduce the fluctuation to 20 per cent or less, at full-load torque.

The reason for this fluctuation in the torque will be apparent if we consider that, as shown on page 30, the torque in synchronous watts is equal to the loss in the rotor, only on condition that the flux does not change either in its distribution or in value during rotation. Thus considering the case considered there, that of a rotating field magnet revolved around the rotor of an induction motor, it was shown that if the field did not vary all of the power imparted to the stator appeared in the rotor as I^2R loss, and since the torque on the field and that on the rotor are equal, the torque in synchronous watts is equal the loss in the rotor.

If, on the other hand, we should keep the field at rest, and excite it by means of an alternating current, it is evident that we should have a large current produced in the rotor and in consequence a large loss in it, but no torque at all would be developed. We have seen that in the case of

a motor operating near synchronism, the rotor acts powerfully to prevent any change in the shape of the flux distribution wave. At standstill this action is nearly absent, and in consequence, even with a sine wave of applied e.m.f., the flux wave may be very much distorted, and what is more serious, may change materially in its space distribution from point to point. This action is seriously increased from the fact that as the rotor revolves, the relative positions of the stator and rotor coils change, and in consequence the impedance of the motor is changing. An ammeter or wattmeter connected in the primary circuit will show the fluctuation of the current or power. This last action is almost entirely absent in a well-designed squirrel-cage motor. The fluctuation in the shape of the flux wave is, however, present to some extent in both the squirrel-cage and in the wound-rotor machine.

It will be readily seen from the above that when a motor is in the act of starting, there exists a rotating magnetic flux which is, however, neither of constant value or of constant space distribution. We may consider roughly that that portion of the flux which does not change acts to produce the torque of the motor. The variable portion sets up currents in the rotor which are largely ineffective in producing torque. In the case of the wound-rotor machine, we have the further fact that the torque in addition to being weakened by the process just described, varies at different points of the revolution, due to the varying relative positions of the rotor and stator.

As is pointed out elsewhere, the use of a fractional pitch winding is equivalent to allowing the different bands of current to overlap. This has the effect of making the fluctuation of the rotating flux at starting less, and in consequence the starting torque will be increased. It might also be pointed out that the single-phase induction motor represents the extreme case of fluctuation of the flux. In fact, at the moment of starting, there is no rotating component, and all of the flux may be considered as being a fluctuating component. The starting torque is therefore zero.

CHAPTER IV

STARTING DEVICES

In starting an induction motor, various devices are used to reduce the current required. In the case of the wound-rotor machine, the starting device takes the form of a three-phase resistor or its equivalent. This usually has such resistance as to allow approximately full-load current to flow when it is all in circuit. The torque developed is then approximately equal to the full-load torque of the motor. The maximum torque that the motor can develop is attained when the value of this resistance is such that the end of the current vector is at nearly the top of the circle. The power-factor of the rotor current is then evidently equal to 70.7 per cent, since the angle of lag is 45 degrees. A decrease of the resistance below the value required to give this current would evidently cause the motor to take more current and at the same time give a reduced torque. Hence it would never be advisable to reduce the resistance on the first contact-point below this value. If the resistor were to remain in circuit permanently the above conclusion would of course require modification, but in this case a squirrel-cage machine would be preferable.

The value of resistance to produce the above torque can be very readily determined from the completed machine. To do this it is merely necessary to connect the motor with a wattmeter, ammeter, and voltmeter in circuit. The secondary resistance can then be varied until the input is a maximum. This will be nearly the point of maximum torque as is at once apparent from the circle diagram. Of course the torque may be measured directly if more convenient. It is also evident that the value of the resistance may be calculated directly, if the circle diagram of the motor is known.

In the case of any except the smallest squirrel-cage motor, a starting device is necessary, not so much on account of the motor, but on account of the circuit and the connected machinery. As far as the motor is concerned, it would not be injured by the large momentary current. In any event the power expended in bringing the load up to speed is wasted. The power to supply this loss is furnished at approximately the same efficiency whether the load is accelerated slowly or rapidly. The power

wasted in friction, on the other hand, will be greater the slower the acceleration. Hence the total amount of heat developed in the motor will, if anything, be less if the motor be thrown directly on the line.

This procedure would, however, in most cases result in drawing such a large current from the line that the voltage of the circuit would be seri-

Fig. 21.—Western Electric Auto-starter.

ously lowered, and hence it would be objectionable. In many cases also, the acceleration would be so rapid that the belts would slip and be damaged or the connected machinery might be injured. The point to be noted is that the starting devices are necessary on account of external actors, and not primarily for the sake of the motor.

The most usual form of starting device for squirrel-cage motors is an auto-transformer to reduce the voltage applied to the motor terminals during the process of starting. After full speed is attained the trans-

former is disconnected from the line. Usually the voltage is applied in two steps, a reduced voltage and the full-line pressure, but in the case of some very large motors the pressure may be applied in several steps. The external appearance of an auto-starter is shown in Fig. 21, and the internal appearance in Fig. 22.

The transformers used in a starter of this type are usually auto-trans-

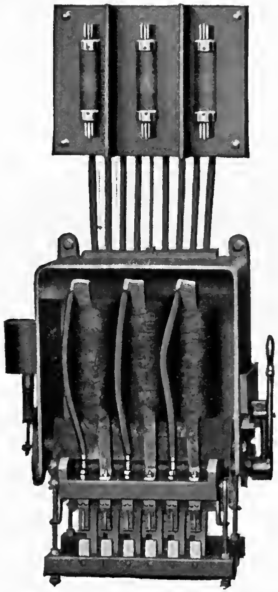


FIG. 22.—Western Electric Auto-starter. Interior View.

formers. The amount of copper required in an auto-transformer is much less than would be required in one having both primary and secondary coils. This is particularly the case when the ratio of transformation is near to one. In the limiting case, when the ratio is one, the power rating of a given transformer is infinite. The truth of these facts can be readily seen from Fig. 23. For simplicity it has been assumed that the transformer has 100 turns in all, and that the secondary is tapped off from 70 turns. If then the primary voltage is 100, the secondary voltage,

neglecting losses, will be 70. Let us assume that the current taken from the secondary is 100 amperes. Since the secondary output is $70 \times 100 = 7000$ volt-amperes, the primary current must be 70 amperes, giving the same volt-ampere input.

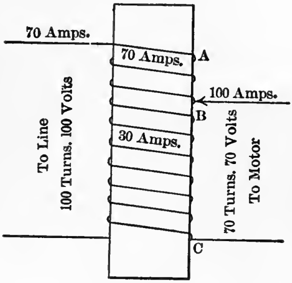


FIG. 23.—Representation of Auto-transformer.

The currents in the various parts of the windings are readily deduced. Obviously the current in the part *A B* is 70 amperes. Since the current taken from the transformer is 100 amperes and since it is the sum of the currents in the windings *A B* and *B C*, the current in the latter must be 30 amperes. If the transformer were provided with both a primary and a secondary coil, the primary would consist of 100 turns of wire large enough to carry 70 amperes, and the secondary of 70 turns capable of carrying 100 amperes. Thus the amount of wire required for the auto-transformer is readily seen to be only 30 per cent as much as would be needed for the type having two coils. In fact it can readily be seen that the amounts of wire required in the two types are in the ratio of $E_p - E_s$ to E_p, where E_p is the primary voltage and E_s is the secondary voltage. Since for starting induction motors the reduction of voltage is rarely more than 50 per cent, the great advantage of auto-transformers is readily comprehended.

Moreover an induction-motor starter is usually provided with several taps, so that a number of secondary voltages can be obtained, depending upon the load to be started. It is clear that at the larger voltages a greater output is taken from the transformer. This is readily provided for in the auto-transformer, since the power capacity is much greater at the larger ratios of transformation. Of course in actually designing an auto-transformer, not all of the saving would be made in the copper as indicated, but the section of iron, as well as that of the copper, would be reduced. Since the apparatus is for intermittent use only, both the copper and the iron are worked at very high densities, and the apparatus is very small compared to its power rating.

For two-phase circuits, two auto-transformers are used, one connected across each of the two phases. In the case of a three-phase starter, there are two types in general use. The one uses a three-phase trans-

former, as shown in Fig. 24. The diagram also shows the connections of the starting switch. This latter is usually of the oil-immersed type, and is generally constructed as an integral part of the starter. The middle row of contacts are mounted on a drum, and can be thrown either up or down so as to connect to either the upper or the lower row of contacts. The connections can be readily traced out and it will be seen that when the switch is in the upper position the motor is connected directly to the line through the fuses, and the transformer is entirely disconnected from the line. In the lower starting position of the switch, the transformers are in circuit, but connected outside of the fuses. This is necessary in order that the fuses may not be blown

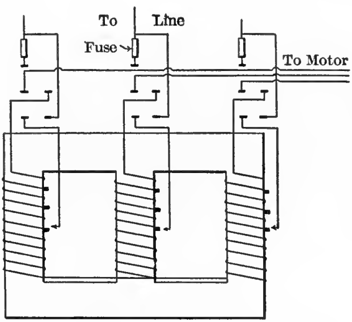


FIG. 24.—Connections of Three-phase Auto-starter.

by the large starting current. Occasionally large starting fuses are used in addition to the running fuses. If this is the case they are connected in the three taps leading from above the running fuses. This is usually not considered necessary, as a short-circuit on an alternating-current circuit is not nearly so destructive as one on a direct-current circuit, on account of the fact that the reactance which is always present cuts down the current. A number of taps are provided on the transformers so that the starting voltage may be varied to give the starting torque with as little current as possible.

Instead of using a three-phase transformer in the way just indicated, many manufacturers use instead two single-phase transformers connected in open delta. This has the advantage of allowing the same transformers to be used for a two-phase motor. Fig. 25 shows the

connections of an auto-starter of this type. This method of connection has the advantages over that of Fig. 24, that a smaller number of contacts are required, and that the heavy starting current of the motor circuit does not have to be broken.

Resistance Starters

In the case of the smaller motors, say those under 30 h.p., many manufacturers use instead of the auto-transformer as above described, a resistance type of starter. Such a device is shown in Fig. 26. The resistors are usually grouped in three equal parts connected in the three phases, and arranged to be cut out in equal steps. This may be done

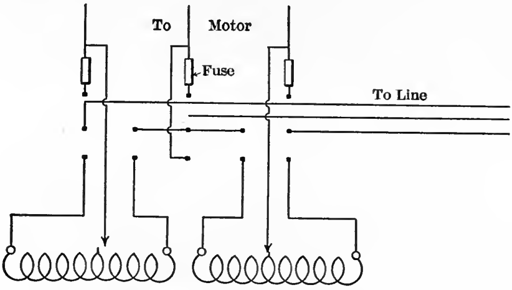


Fig. 25.—Connections of Three-phase Auto-starter.

in much the same way as in the case of the direct-current motor, by dividing the resistors into many parts which are successively cut out of circuit, or the number of steps may be made much less than is common with direct-current motors. In fact, as far as the motor and circuit is concerned, the resistance may all be cut out in one step, without causing any more disturbance than would be the case if an auto-starter were used. The connections must be so made that the fuses are out of circuit during the starting period, the same as in the case of the auto-starter.

There are two other methods of starting which deserve notice. In some cases it is possible to bring out taps from the supply transformers, giving a reduced voltage for starting. In this case a three-pole double-throw switch may be used. Care should be taken to see that the switch is of ample size to carry and break the excessive starting

current, or bad burning of the contacts will result. The principal difficulty in applying this method is the cost of running the two or three extra wires required, and the fact that standard transformers are not provided with suitable taps. In cases where half voltage is sufficient, standard transformers can frequently be used. In this case, however, since the starting torque varies as the square of the voltage, the starting torque will be only one-quarter of the maximum the motor can develop, and this would usually be insufficient.

Another method sometimes made use of is to connect the motor windings in star for starting and in delta for running. Since the

FIG. 26.—Fairbanks Morse Resistance Type Starter.

voltage over one phase when the motor is connected in star, is the line voltage divided by the square root of three, the starting torque will be only one-third of the maximum. This would frequently be insufficient, and the method lacks flexibility, since this torque cannot be changed. It is also usually out of the question, since most motors are connected in star rather than in delta, for normal operation. It is, of course, possible to develop a complete line of motors with delta windings, and use the star delta method of starting in all cases where it will suffice, resorting to one of the usual types of starter when necessary. This has been done by the Crocker-Wheeler Company, of Ampere, N. J.

Another method of starting used by the Richmond Electric Company of Richmond, Va., is illustrated in Fig. 27. In addition to

the usual winding of the motor, additional turns are wound on the stator and connected in series with the stator winding. For starting, all of the turns are used. For running, connection is made to the terminals of the stator winding, and the starting coils are left on open circuit. With all of the coils in circuit, the motor is equivalent to a motor of smaller rating, and of course, starts with a reduced current. The motor might in fact be operated indefinitely with the switch on the starting position, provided the load were not too great for the reduced rating.

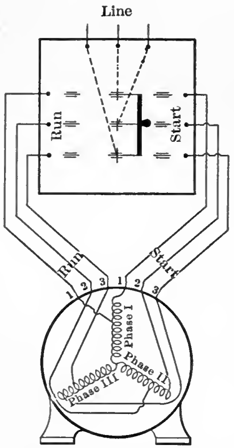


Fig. 27.—Connections of the Richmond Induction Motor.

The advantage of the method is that it does away with the need of the external starter, and permits of a simpler starting switch, since the switch need be only three-pole on both sides. On the other hand, in order to accommodate the greater number of turns, it is necessary that the slots be deeper. This in turn requires a greater diameter of the stator punchings, and consequently a larger motor. The leakage factor is also slightly increased, which causes the power factor to be slightly lower.

It should also be mentioned that it is frequently possible to effect a saving by arranging to start several motors from the same starting device. Fig. 28 shows the connections for doing this. A set of starting wires must be run to each motor in addition to the usual supply wires, and each motor must be provided with a triple-pole, doublethrow switch. This switch is thrown to the lower position, and the motor started in the usual way. The switch is then thrown to the upper position and the starter returned to the off-position. In some cases when motors are slightly loaded for long period, it might be advisable to replace the starter with a transformer, left constantly in circuit. Motors carrying light loads might then be left running on the lower voltage. The power-factor would thereby be very materially raised and the losses lessened. Unless the motors are very close together, the expense of wiring in the manner shown is apt to be greater than the cost of the usual starters. The operation of starting is also more com-

plicated, and the apparatus is more apt to be damaged by careless handling.

It should be noticed that if an auto-transformer is used, any motor not of greater rating than the starter may be started from it. Thus it would be entirely proper to start a 10 h.p. motor from a 100-h.p. starter. This is not true in the case of the resistance type of starter unless the range of resistance is very great. However, the only damage done would be that if a starter of a larger rating were used, more starting current than necessary would be taken from the circuit, and if the acceleration were too rapid, damage might be done to the belts or connected machinery.

In the case of motors of 5 h.p. and less, it is customary to throw them directly on the line. Frequently, however, switches with a double set of clips are used, so that the fuses are cut out during the operation

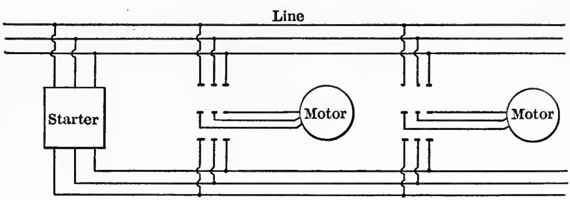


FIG. 28.—Connections for Starting Several Motors with One Starter.

of starting. In this case, the switch should be so designed that it cannot be left on the starting position.

The question of the relative advantages of the auto-starter and the resistance starter is of importance. There has been a disposition on the part of some central-station managers to object to the resistance starter on the ground that it takes more current and therefore interfered more with the regulation of the line than the auto-transformer. It can be readily shown that this is not the case, and in the writer's judgment the balance of advantage lies on the side of the resistance starter for small motors, and on the side of the auto-starter for large ones.

There are five important qualities that a starter should have. These the writer would place in the following order: Minimum line disturbance, minimum power consumption during starting, self-adjusting properties, i.e., the starter should automatically raise the voltage over

the motor terminals as the motor speeds up, it should be readily adjustable for different starting torques, and it should be low in cost.

In considering line disturbance, it should first be noted that it is not the power component of the current that is principally responsible for the line drop. The drop is due almost entirely to the wattless component of the current. To go into this fully would require a discussion of the regulation of generators, lines, and transformers. It is a fact, however, that in all of these cases, the drop of pressure is due almost entirely to the component of current lagging 90 degrees. Thus in the case of a typical alternator the regulation at unity power-factor might be 6 per cent, while at zero power-factor, it might be 50 per cent or more. The comparison would be even more unfavorable if we took the drop in voltage when full-load current was thrown on, first at unity, and then at zero power-factor. The same thing is true of transformers and transmission lines. For all practical purposes we may almost entirely neglect the drop due to the power component of the current and consider only the wattless component. We shall now proceed to show that this wattless component is the same, whether the resistance type or the auto-starter is used.

Let E_l=line voltage, E_m=the voltage over the motor terminals, and I_L and I_m the line and motor currents respectively, and R and R_s the resistance of the motor and of the starting resistor. Then in the case of the auto-starter we have

$$I_m=\frac{E_m}{\sqrt{R^2+L^2\omega^2}}.$$

The wattless component of the current is

$$I_{mw}=I_m \sin\theta=\frac{E_mL\omega}{R^2+L^2\omega^2},$$

and if α be the ratio of transformation,

$$\alpha=\frac{E_m}{E_L} \quad \text{and} \quad I_{mw}=\frac{I_{Lw}}{\alpha},$$

then the wattless line current is

$$I_{Lw}=\frac{\alpha^2E_LL\omega}{R^2+L^2\omega^2}.$$

Similarly the power component of the line current is

$$I_{Lp}=\frac{\alpha^2 E_L R}{R^2+L^2\omega^2},$$

and the power is

$$P=I_{Lp}Ei=\frac{\alpha^2 E_L{}^2 R}{R^2+L^2\omega^2}.$$

In the case of the resistance-type starter, let us assume such a value of resistance that the voltage over the motor, the motor current and consequently the torque are the same as before. The motor and line currents are now obviously the same, then

$$I_m=I_L=\frac{E_m}{\sqrt{R^2+L^2\omega^2}}=\frac{\alpha E_L}{\sqrt{R^2+L^2\omega^2}}=\frac{E_L}{\sqrt{(R+R_s)^2+L^2\omega^2}}.$$

The wattless component of this current is

$$I_{Lw}=\frac{E_L L_w}{(R+R_s^2+L^2\omega^2)}.$$

Substituting in the parenthesis from the previous equation,

$$I_{Lw}=\frac{\alpha^2 E_L L\omega}{R^2+L^2\omega^2}.$$

Similarly the power component of the current can be shown to be,

$$I_{Lp}=\frac{\alpha^2 E_L (R+R_s)}{R^2+L^2\omega^2}.$$

and the power is

$$P=\frac{\alpha^2 E_L{}^2 (R+R_s)}{R^2+L^2\omega^2}.$$

In the above the wattless components of the line current are seen to be the same in the two cases, while the power components are in the ratio of the motor resistance to the resistance of the motor and the starter. The motor resistance used above is of course the equivalent resistance of the stator and the rotor combined. We thus see that the component of the current, to which the drop is principally due, is the same in the two cases. The power component of course produces some drop, and this is greater in the resistance type starter, but this is largely, if not entirely offset by the extra lagging current required to

magnetize the transformer, and to produce the leakage flux of the latter.

The writer has tested the above deduction on a number of motors and starters. In no case was the disturbance to the line voltage materially different in the two methods. If anything the balance was slightly in favor of the resistance starter.

As regards the second requirement, minimum power consumption during starting, the auto-transformer has an undoubted advantage, since all the energy expended in the starting resistor is wasted. The time of starting is, however, usually so short that the value of this wasted energy is not of much moment. In cases, however, where the motor is of considerable size compared to the generator, the power requirement during starting may be so great as to overload the generator and engine, and make the use of the auto-starter necessary or desirable. In present-day practice the line is usually drawn at about 25 h.p., but there is no reason why with large generators, larger motors should not be started with the resistance type starter.

In the third requirement, (i.e., self adjustments of voltage) the resistance starter is clearly superior. This is due to the fact that as the motor speeds up, the current decreases, due to the counter e.m.f. increasing, and consequently the voltage applied to the motor terminals increases automatically. This means that when the motor is finally thrown directly on the line, the sudden change in voltage will not be so great and consequently the supply voltage will not be disturbed to so great an extent as with the auto-starter. If the starter is supplied with a number of intermediate steps, the disturbance is of course still less.

In regard to the feature of ready adjustability, the resistance starter, if it is provided with a number of intermediate steps, is evidently superior, since it is inherently self-adjusting. It should be set to give on the first point about 70 per cent of full-load torque. If more than this is needed, the attendant at once secures it by moving the starting arm to the required notch. Thus no more current is taken than is absolutely required. This feature is particularly valuable in cases where the starting torque is apt to vary widely at different times.

It is evident that as regards the question of cost, the resistance starter can be manufactured at a lower cost than the auto-starter. This fact, together with some of the points brought out, will, it is believed, insure a large use for it, at least in the smaller sizes.

Carbon Block Starters

Since the foregoing was written, a new type of resistance starter for induction motors has been put on the market by the American Electric Fuse Co., of Muskegon, Mich. Starters embodying the same principle are also built for use with direct-current motors. The essential feature of this device is the employment of the imperfect contact between carbon blocks to produce the resistance. It is well known that if two carbon blocks are pressed together with a small force the resistance to a current will be large, and if this force is increased the resistance will be very much reduced. It is possible in this way to obtain a variation of resistance of as much as 200 to 1, without employing such small pressures as to render the contact uncertain. The same principle has been utilized for a number of years in various devices, notably in the telephone transmitter.

Considered from a theoretical standpoint, this construction seems to offer a number of advantages. Of these, perhaps the most apparent is the fact that all of the resistor is in use all of the time. Each resistor unit has a definite rating in watts, the value of this rating depending, of course, on the nature of the service, i.e., whether intermittent or continuous. It is entirely independent of the value of the current or of the e.m.f., and depends only on their product. Thus one unit can be used for a variety of different currents and e.m.fs., providing only that the rated power in watts is not exceeded. This is not true of the more usual form of rheostats, since if the current is increased by cutting out a portion of the resistor units, the watt rating of the apparatus is reduced, because the units cut out are then not available for radiating the energy lost in the resistance.

From the standpoint of the user, this means that it is possible to start the motor whether a.c. or d.c. with the minimum amount of current, and consequently with the minimum chance of injury to the connected apparatus. From the standpoint of the electricity supply company, it means that the motor demands during the starting period, only the minimum amount of current and power that will possibly start it. We have shown in the case of the polyphase induction motor, that the amount of wattless current is the same whether a resistance type or a transformer type starter is used. Since in the case of a starter of the kind just described, only the minimum amount of starting current is used, it follows that with a carbon-block starter the amount of a wattless current will in general be less than in the case of the trans-

former starter, and consequently the line disturbance will in general be less. In the limiting case, when the transformer starter is so set that it can just start the load, the wattless current and the line disturbance will be the same.

The other advantages mentioned in connection with the resistance-type starter apply in this case also. A perspective view of one of these starters is shown in Fig. 29. It will be noted that there are six connectors at the top of the starter. The three extra terminals are for the

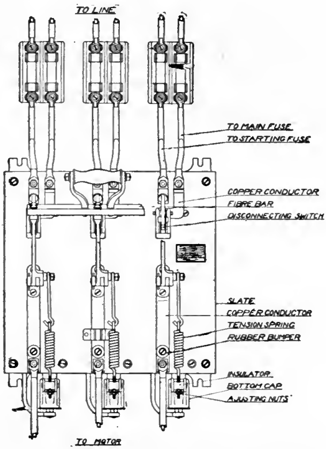


FIG. 29.—Connections of American Electric Fuse Company's Starter.

insertion of the starting fuses or to allow the lines to be connected back of the fuses for starting. During the time of starting, the running fuses are short-circuited by the three small levers shown. Fig. 30 shows the curves of time and power during the starting of a 15-h.p. 440-volt three-phase induction motor with a carbon-block rheostat and with an auto-starter. The motor was started under a heavy load consisting of a line shaft and counter-shafting. The load was of course the same in the two cases. The auto-starter was one that would show up to the best advantage, since it was of the three-point type, thus affording smoother acceleration than would have been the case if the ordinary type of two-point starter has been used. The high peaks of power as

the auto-starter was thrown successively to the three starting positions are very apparent. In the case of the carbon-block starter, on the other

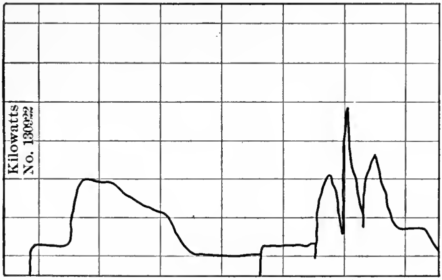


FIG. 30.—Starting Currents of an Induction Motor with American Electric Fuse Co's. Carbon Block Starter and with Three-point Auto-starter.

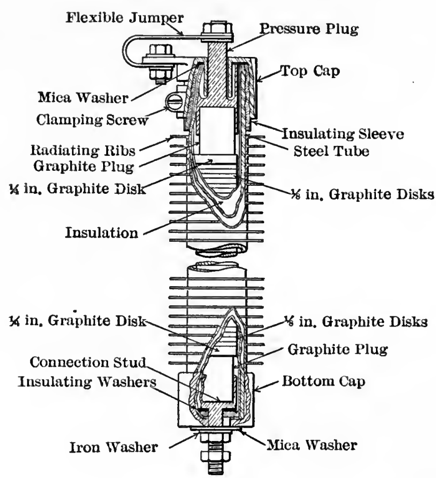


FIG. 31.—Section of American Electric Fuse Co's. Resistance Unit.

hand, the power taken can be kept nearly uniform, and the sudden jumps in power and consequently in the voltage can be avoided. In

Fig. 31 is shown a section of one of the resistance tubes. From this the construction can be clearly seen. As will readily be surmised, the principal difficulty in the development of this piece of apparatus, was the production of a suitable lining for the iron tube used to contain the disks, since it is necessary that these be insulated from the rest of the starter. Given a suitable lining, the apparatus is almost indestructible. The writer understands that this lining has been brought to a high state of perfection.

Protection against Overloads in Induction Motors

The provision of proper protection (for the squirrel-cage induction motor) is a more difficult problem than in the case of the direct-current machine. This is on account of the fact that the squirrel-cage induction motor requires a much greater current during the starting period than it does while in normal operation. The starting current may amount to as much as five times the running current and a fuse which would afford protection during starting would be of little or no use during ordinary operation. Owing to the great difficulty of arriving at a satisfactory solution, the National Board of Fire Underwriters have apparently hesitated to make rigid rules which might hinder the develop- of the industry. As a consequence, the regulations are not nearly so rigid as is the case with direct-current motors. Thus the practice of starting with no fuses in circuit is at least tolerated, and a size of wire supplying the motor corresponding in current-carrying capacity to the starting current of the motor is rarely required.

The provision requiring a no-voltage release on all direct-current motor starters is not enforced in the case of the alternating-current motors. The object of the release is that if the power is temporarily removed from the line the starter may return to the off-position. When the power is again restored, the motor will remain disconnected until properly started by an attendant. The induction motor would, however, not be damaged by having the power thrown on under such circumstances, and about all the harm that would be done would be the blowing of the fuses. Hence, it is doubtful if such a provision would be warranted.

The use of fuses in connection with any motor is open to some serious objections, and the principal point in their favor is their low first cost. If used under circumstances where they are frequently blown, the cost of renewals may be a very considerable item, and may render the use of a circuit-breaker preferable.

When used with induction motors, there is an objection to the fuse which does not apply in the case of direct-current machines. In the latter case, if one of the fuses is burned out the motor will not start, or if in operation, it will at once stop. In the case of the polyphase induction motor, since the motor does not take its starting current through the running fuses, one (or more) of the latter may be burned out and the motor will start just as though nothing were the matter. After the motor has attained nearly full speed, the starter handle will be thrown to the running position, and if two of the fuses of a three-phase motor, or the two fuses of one phase of a two-phase motor are 'ntact, the motor will continue to operate as a single-phase machine. This condition may readily escape notice, since the motor appears to operate normally. If the fuses are inspected the indicator used with enclosed fuses may have failed to operate and the fuse will consequently be apparently all right. Even though a test is made, some electricians will test with a voltmeter over each of the fuses while the motor is in operation. There will, of course, be little or no voltage over the defective fuse, and this test will show everything apparently all right. It may appear that undue emphasis is laid upon a remote possibility, but any engineer who has had much to do with induction motor troubles can testify that cases of this nature very frequently occur.

When this condition is present, the winding of the motor still in circuit is apt to be overloaded and damaged. It might appear that on account of the overloading of the remaining phase, the fuses on this phase would open and relieve the motor. This, of course, often happens, but in many other cases, it is found that advantage has been taken of the sturdy character of the induction motor and heavier fuses than should normally be used have been substituted. This may have been done on account of the trouble frequently experienced by fuses blowing when the starter is thrown from the starting to the running position.

By using a circuit-breaker, the possibility of this occurrence can be almost entirely prevented. To take care of the starting period, taps can be brought out of the trip coils, so that only a portion of the turns are in circuit while the motor is being started. This can be done at a small expense, and affords at least partial protection during starting. The rush of current at the instant when the starter is thrown from the starting to the running position, is more difficult to deal with. Some provision may be made by which the attendant may temporarily short-circuit some of the turns of the trip coils, the device being so constructed that it will immediately throw these turns in circuit as

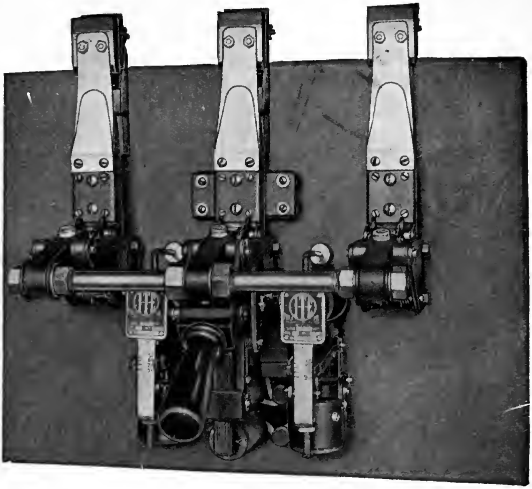


FIG. 32.—I.T.I.E. Time Limit, no Voltage and Overload, Circuit Breaker, Built by the Cutter Co.

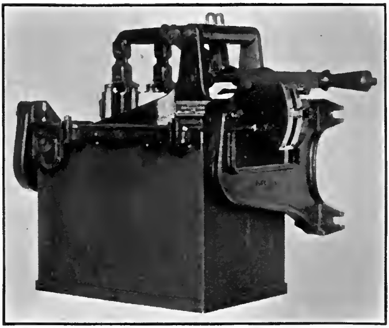


FIG. 33.—Westinghouse Auto-starter Switch.

soon as the attendant leaves the starter. Several other solutions along the same line have been proposed.

One of the devices which seems most promising is the use of a time-limit relay in connection with the circuit-breaker. With the addition of this device, the breaker will not at once open on the application of the overload, but it will be necessary that the condition of overload continue a definite time, before the breaker will operate. This is of some advantage in the normal operation of the motor, since it is not necessary to set the breaker for a large current, in order to take care of harmless temporary overloads. It is, however, of the greatest benefit in taking care of the starting period in a satisfactory manner.

A breaker of this type manufactured by The Cutter Company, and adapted for use on circuits of 550 volts and under is shown in Fig. 32. A Westinghouse switch adapted for use on 2200-volt circuits is shown in Fig. 33. In this case, the switch is of the double-throw variety with six contacts on one side and three on the other in the case of a three-phase outfit. The same switch will then serve both as a starting switch and as a circuit-breaker to protect the motor in regular operation. The use of such combined breakers and starting switches on 2200-volt circuits is quite common, but on account of the cost and the recent introduction of the apparatus, the fuse protection is more usual on low-voltage circuits.

With high-voltage outfits, the use of oil as a medium to assist in breaking the arc is almost universal. Most of the makers have followed the same practice in the case of the low-voltage breakers. This has probably been largely due to the fact that the motor itself having no sliding connections is free from the possibility of sparking and the attendant fire risks. This is of importance in certain industries, and naturally leads to the wish to obtain the same desirable property in the protective device. In places where an open arc can do no harm in the event of the breaker opening, there is apparently no reason why the breaker with the break in air should not be equally reliable. Certain incidental advantages such as the removal of the possibility of the oil leaking out without being noticed, and being absent when most wanted, are secured by the air break. The fact that it is at once apparent whether or not the breaker is open, is also of some advantage.

The above remarks apply in general to the squirrel-cage type of motor. The wound-rotor machines can be handled along the same lines found desirable with direct-current machines, since the starting current is not necessarily much greater than the running current.

CHAPTER V

THE INDUCTION GENERATOR

ANY induction motor may be operated as a generator. In Fig. 34 is shown the circle diagram of an induction motor, but extended to include the action of the machine as a generator. The line *OB* represents in magnitude and phase the current taken by the machine when

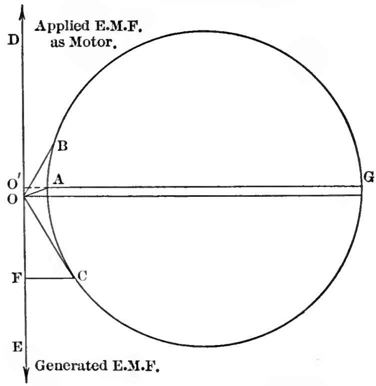


FIG. 34.—Circle Diagram of Induction Motor or Induction Generator.

acting as a motor under approximately full load. The line *OA* represents the current at synchronism. If now the motor be forced to run at a speed in excess of synchronism, the current vector will be represented by some such line as *OC*. A component of the current is now in the same phase as the generated or counter e.m.f. or the machine is acting as a generator.

That this will be so is also readily seen from a consideration of Fig. 7, page 6. It will be apparent that if the rotor be forced to revolve at a greater speed than synchronism, the rotor conductors will cut the flux as before, but in the opposite direction. The e.m.f. gen-

erated and consequently the current will be reversed in the rotor, and this will require a power component of current in the stator in the reverse direction to offset it. Since the current in the rotor is reversed, the torque is also reversed, or the machine acts as a generator.

Several things will be at once apparent from an inspection of the diagram. The maximum output as a generator will not be so large as the maximum input as a motor. If this output is exceeded, and the torque applied to the motor is maintained, the machine will speed up indefinitely. Of course in practice this would not be the case, as the governor of the prime mover would act to limit the speed. It is also evident from the diagram that the maximum power-factor will be slightly less as a generator than as a motor.

An induction generator of this type is not self-exciting. Both its voltage and its frequency are fixed by the voltage and frequency of the line to which it is connected. The power it delivers is determined by the amount by which it exceeds synchronous speed. If it were desired to operate a station using only induction generators, it would be necessary to provide a sufficient capacity in synchronous machines to supply the magnetizing current of the induction generators. The combined capacity of the synchronous machine would therefore need to be approximately 25 per cent of the capacity of the induction machines, provided the power-factor of the connected load were unity. The exciters could take the form of synchronous motors, running idle on the line, or they might be of somewhat greater rating, and be used to supply a certain amount of power to the station busbars, or be used as motors, in addition to furnishing the magnetizing current of the induction generators.

It will be seen that the current supplied by the induction generator is leading, considering the machine as a generator. That this is so will be apparent from Fig. 34. The line OD represents the phase of the applied e.m.f. when the machine is running as a motor. The back e.m.f. of the machine is represented by the line OE, and when the machine is acting as a generator, this becomes the terminal e.m.f. The direction of rotation of the vectors was taken as counter-clockwise, and consequently the current now leads the e.m.f.

A possible arrangement of the apparatus is shown in Fig. 35. We have here an induction generator electrically connected to a synchronous machine, and the combination connected to the line. The two machines must not be mechanically connected, but must be free to rotate at the proper speed to give the required slip to the induction machine.

Suppose for the minute that there is no load on the line, and that the synchronous machine is not mechanically driven, but acts only as a synchronous motor. In order to supply its losses, the synchronous machine will require a small component of power current, and the induction machine will require a much larger component of wattless current to magnetize it. This current is leading the e.m.f. of the induction generator. Consequently the synchronous machine must supply a lagging current considered as a generator, or take a leading current considered as a motor. Its current is thus almost entirely wattless, and its field must be strengthened beyond the point that would be necessary if it were operating at unity power-factor. We may, in fact, consider that in a sense the field excitation of the synchronous

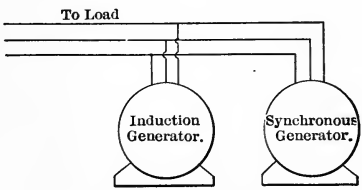


Fig. 35.—Connections of Exciter and Induction Generator.

machine must be sufficient to force the required flux across the air-gaps of both machines.

If now a load be connected to the generator, the conditions may be materially changed. The character of the connected load determines the nature of the current supplied. If the load, for example, consists of incandescent lamps only, the power factor will be practically, unity. If it consists of induction motors, the current will be lagging, or if, on the other hand, the load consists of synchronous motors with overexcited fields or of a non-inductive load at the end of a long transmission line, the current may be leading. The induction generator, it will be noted from Fig. 34, can furnish current at only one power-factor for each value of the load. This current consists of a component in phase with the e.m.f., and a component leading it by 90 degrees. This leading component is nearly constant, but increases somewhat as the load increases. If the connected load takes a current having a wattless component differing from that which the generator can supply, the excess or deficiency of wattless current must be furnished by the

synchronous machine. Thus if the current supplied by the generator be represented by OC, it consists of a power component OF and a wattless leading component FC. The power-factor of the generator current is OF divided by OC. If the load demands a current of this same power factor, the synchronous machine will supply no current. This neglects of course the small losses of this machine.

If on the other hand the circuit demands a lagging current, the synchronous machine must supply not only the leading current required by the induction machine, but also enough leading current to bring the power-factor of the whole output to that value of the power-factor which the generator is capable of furnishing at the load considered. In general this current will be considerable, since in the great majority of cases the current required is lagging, and in this case the synchronous machine must furnish not only all the lagging current for the load, but in addition the current required to excite the induction machine.

The question will arise in the reader's mind, since a leading current is required for excitation, why is it not practicable to make the induction generator a self-excited machine by providing a condenser to consume the leading current required? This could be done without an adjustable condenser, although the cost would be very great, providing the power-factor of the load to be supplied did not change. What would be required in a practical case, would in fact, be a combination of a condenser and an inductance, both capable of being adjusted to any value required, and of sufficient size so that the power-factor of the load, combined with the regulating inductance and condenser, could always be brought to that value of power-factor which the generator was capable of supplying under the given load. It will be seen at once that if the load were of such a character that the power-factor were at all variable, the size and cost of such a regulating device would be prohibitive.

Such a combination would also be somewhat unstable in voltage, since the only factor which would tend to hold the voltage constant is the fact that the power-factor of the induction generator changes slightly with a change of voltage. This is true since the ratio of the magnetizing current to the locked current changes somewhat with change of voltage on account of the saturation of the iron. This increases the magnetizing current for greater voltages in a greater proportion than it increases the locked current. Since the maximum power-factor is equal to $\frac{1}{1+2\sigma}$, and since $\sigma=\frac{O'A}{AG}$ it will be seen at once that for each value of the

power-factor of the load, the generator would tend to assume a certain voltage. This voltage would, however, not be as definite as might be desirable, since the iron of the induction motor is not in general worked at such a density as to reach the knee of the magnetization curve. It would be more definite on 25 cycles than on 60 cycles, since in the former case higher densities are employed than in the latter.

The arrangement using a synchronous machine for excitation is likewise open to serious objection, on account of the cost of the two machines, and on account of the poor regulation of the combination. It is evident that by increasing the size of the synchronous machine somewhat, providing it with a prime mover, and using it to supply a part of the power required, that the cost per k.w. of output will be reduced. For instance if the synchronous machine were furnishing 100 k.v.-amp. of wattless current, by increasing its size to 141 k.v.-amp. it would be capable of furnishing the same wattlers current as before, and at the same time it could carry a power component of current corresponding to an output of 100 k.w. This follows since the two currents are 90 degrees apart in phase and their resultant is therefore equal to $\sqrt{2}=1.41$ times as great as either. We should therefore obtain a power rating at 100 k.w. at the cost of an increase in size corresponding to 41 k.w. It is doubtful, however, whether the conditions would ever be such as to justify an installation of this kind.

In the case of a plant already installed, to which it is desired to add another machine, the case against the induction generator is not so strong. Consider for example a large plant delivering its energy through a long transmission line, and suppose it is desired for some reason to feed into it from a generator driven by a gas engine. Owing to high frequency or the great resistance and reactance of the lines between the main generators and the gas engine set, hunting of the latter unit may be feared. The use of an induction generator would largely remove this danger. In such a system, moreover, it is entirely possible that the current taken by the line and load may normally be leading, on account of the capacity of the line. In this case the induction generator, since it gives a leading current, will supply at least a part of the leading current required, and hence will relieve the main generators. Some difficulty will be experienced if an attempt is made to govern the gas engine, since it would be necessary to use a governor such that the speed would increase with the load. Probably the most satisfactory method of operating would be to run the gas engine at constant load, merely providing a hand throttle and a speed limiting device to prevent the running

away of the engine in case the breakers should open and remove the load from the generator.

A similar problem is sometimes encountered in the case of small water powers. It frequently happens that it would be feasible and cheap to develop such powers, but the expense of an attendant would more than offset the value of the power generated. In such a case we could make use of an outfit consisting of an induction generator, either direct connected or belted to a water wheel, and provided with suitable means for preventing the speed rising much above synchronism in case the power goes off the line or the circuit-breakers open. Such a plant should give satisfactory service with only a daily inspection. It is of course necessary that the rating of the induction generator should be small in comparison with that of the synchronous generators, and if it is not provided with a governor, it is essential that the power required on the line should never be less than the rating of the induction machine.

Another and one of the most promising of the uses to which the induction generator has been put, is in connection with turbine-driven direct-current generators. It is well known that the construction of such a generator presents serious difficulties. These arise principally from two causes, the difficulty of constructing an armature and commutator which will safely withstand the great strains due to centrifugal force, and the difficulty of commutating the current. It is impossible in large machines to keep the voltage between commutator bars down to a reasonable value, since at least one-half a turn must be included between bars (and to do even this requires a rather awkward construction with taps extending from the commutator to the rear of the coil), and the voltage generated in even one-half turn is frequently far in excess of a reasonable value.

It is obvious that a substitute may be found by employing a synchronous alternator, and rectifying the current by means of a rotary converter. This scheme, while perfectly workable, has certain disadvantages. Since both machines are of the synchronous variety, trouble may be experienced on account of hunting, although no serious difficulty is apt to develop. A certain amount of skill and care are necessary in synchronizing the machines, and there is always the possibility of damage being done, due to lack of care in this respect. The most serious difficulty, however, is encountered in the regulation of the voltage. In order that there should not be large wattless currents circulating between the generator and converter, it is essential that the field excitation on the two machines should be such that they would give approximately the same voltage on open circuit. In order to preserve this equality, it

would probably be advisable to provide some form of mechanical connection between the two field rheostats. Moreover, it would be impracticable to compound the converter unless reactance coils were used between the converter and the generator. If this were done the converter would operate at unity power-factor at only one load.

Most of these difficulties are removed by using an induction generator in place of the synchronous machine. To start the set it is merely necessary to run the induction generator at any convenient speed, run the converter by some outside means up to approximately the corresponding frequency connect the two together, and excite the field of the converter. In fact it would probably be preferable to consider the converter as part of the generator and provide permanent electrical connections, that is, have no switches between the generator and the converter. It would also be preferable to connect the generator for a large number of phases, say six or more. This will greatly reduce the copper loss of the converter, or conversely the rating of the converter can be greatly increased for a given size of armature, by simply providing a large enough commutator to take care of the increased output.

In the last few years cases similar to the above have arisen, in connection with certain central stations in which the generating units were driven by reciprocating engines. A reciprocating engine is excellently adapted to utilize steam efficiently in the range of pressures between the usual boiler pressure and the pressure of the atmosphere. The turbine, on the other hand, is better adapted to handle the steam in the range between atmosphere and zero pressure. Moreover, in many cases it was found that it was possible to double the rating of the station by installing a turbine between each pair of reciprocating engines, and still leave sufficient room for the operation of the plant.

If the plant is to operate in this way with each of the reciprocating engines exhausting into one of the turbines, it is essential that one engine and its corresponding turbine be regarded as a unit. The turbine is operated without a governor, taking all of the steam exhausted by the engine.

If the generator operated by the turbine were a synchronous machine, it will be seen at once that the combination might give trouble in various ways. The use of an induction generator in such a combination offers numerous advantages. A notable example of such an installation is afforded in the case of the station of the Interborough Rapid Transit Co. of New York City. The power house contains (9) units each of (7500) k.w., and as noted each unit consists of a reciprocating engine exhausting

into a low-pressure turbine. The generators used on the reciprocating engines are of the usual synchronous type, but those used in connection with the turbines are induction generators. The induction generators are connected to the synchronous machines by simple knife switches, and these switches are not intended to be used in normal operation. To start a set, the field of the synchronous generator is first excited, and steam is then admitted to the engine. The induction machine starts as an induction motor until steam reaches it, when it automatically begins operation as a generator.

Phase Converter

It will be readily seen that an induction motor might be wound for two or more different phases. Thus a machine could be wound with twelve sets of coils per pair of poles and be connected as in Fig. 36. If connection were made to points *AB* and *C*, the machine would operate as a three-phase motor. If points *AHF* and *I* were used it would be two-phase; if *AEBFC* and *G*, six-phase; while if connection were made to all twelve points the machine would be twelve-phase. If operated on any of these numbers of phases, a rotating magnetic field would be set up. Each of the coils would therefore generate a counter e.m.f., the phase of which would depend upon its position on the stator, and there would exist a three-phase e.m.f. at the three-phase terminals, a two-phase one at the corresponding points, etc. No matter what the number of phases of the current applied it would be possible to take off two, three, six, or twelve-phase current from the appropriate terminals.

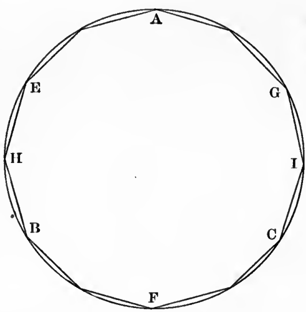


Fig. 36.—E.M.F. Relations of Various Coils of an Induction Motor.

In general, it would not be advisable to use an induction machine in this manner to transform the number of phases, since for this purpose a number of transformers, connected in well-known ways, would be cheaper and better. In two cases, however, this arrangement may be advantageous. It can be readily shown that it is not possible by any

combination of transformers to change from a polyphase system to single-phase, and have the currents of the polyphase system balanced. The energy will in all cases be supplied as single-phase energy from the generator. In general the power in single-phase circuits falls to zero four times in each cycle. The power supplied by the polyphase line must likewise fall to zero, unless some means of storing energy in the system is provided. The kinetic energy of the rotor of an induction motor, used as a phase convertor, supplies a means of storing this energy. The angular velocity of the rotor is not constant, but varies during the revolution. The machine takes energy in approximately equal amounts from all the phases of the supply system. When no energy is demanded by the single-phase system, the motor is being accelerated, and energy is being stored as kinetic energy in the rotor. During the parts of the cycle when the energy demand of the single-phase system is heaviest, the rotor is retarded and gives up a part of its kinetic energy.

The induction machine may also be used in the opposite manner to transform from single-phase to any polyphase system. When operating near synchronism on a single-phase sytsem, provided the motor has a low-resistance rotor, a rotating magnetic field will be set up, and the magnitude of this field will be nearly constant in all positions. Hence a polyphase e.m.f. will exist at the terminals of the corresponding windings. The voltages of the polyphase windings will not be exactly balanced under load, but will be nearly so.

An example of an application where this principle might be used to advantage is in the case of a plant where it is desired to introduce motors to drive the machinery, and only single-phase energy is available. Of course this condition might be met by the use of single-phase motors. As is well known, however, such motors are very costly, compared with polyphase motors of the same rating. Moreover, in many cases there is a strong possibility that the supply will later be changed to polyphase. Under these circumstances, it might be well to install standard three-phase motors, starters and wiring. The three-phase supply lines could then at any future time be connected directly to the wiring, and the plant operated three-phase.

During the time when single-phase energy alone was available, it would be necessary to add a phase-splitting device as indicated in Fig. 37. This can be constructed as shown by connecting a resistor and an reactor in series. It need only be large enough to start one of the three-phase motors unloaded. It may then be disconnected from the line, and

the one motor in operation will generate such an e.m.f. as to cause the third wire to assume the proper phase relation to the other two, and form with them nearly a true three-phase system. The other motors may then be started as three-phase motors.

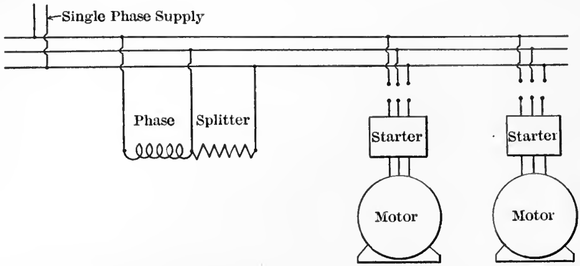


FIG. 37.—Connections for Operating Three-phase Induction Motors on Single-phase Circuits.

A peculiarity of this system is that while the pull-out point of all the motors if loaded each in proportion to its rating, would correspond to the single-phase rating of the motors, the pull out point of any single motor, provided the other motors are lightly loaded, is practically the same as its pull-out point on a three-phase circuit. This property may be of

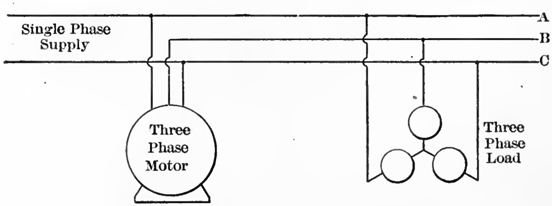


FIG. 38.—Connections of Phase Converter.

great value in cases where the motors are subject to heavy momentary overloads.

To test this action of an induction machine, connections were made as in Fig. 38. A balanced three-phase load was applied to the three-phase line and the voltage between the lines measured for various values of the current. The results are plotted in Fig. 39. The motor used

had a three-phase rating of 7½ h.p., 60 cycles three-phase, 1200 rev. per min. 220 volts. Its full-load current, operated three-phase, was approximately 21 amperes.

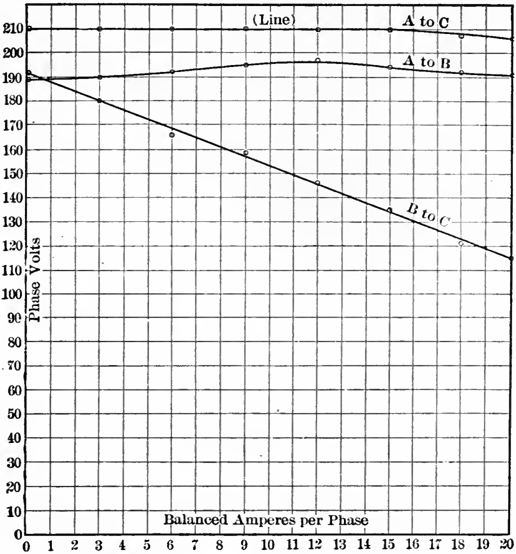


FIG. 39.—Curves of Regulation of Phase Converter.

USE OF AN INDUCTION MACHINE AS A VOLTAGE BALANCER

As a corollary of the use just described, it will be readily seen that if an induction machine be operated from a polyphase line, the voltages of which are unbalanced, that it will act to restore to a certain extent the correct voltage relation. In fact the case just described is merely an extreme case of unbalanced voltage. We may consider the system as a three-phase system in which the voltage between either of the two outside lines and the third line is indefinite; that is, it may be at the potential of either of the other two lines. By starting the induction machine, it is caused to assume a certain definite phase relation to the other two lines.

In a similar way, if an induction machine is operated on an nnbalanced circuit, it will tend to take most of its energy from the phases the voltages of which are high, and little or none from those which are low- In fact, it may readily happen that it even returns energy to the low-voltage circuits, and of course takes an extra amount of energy from the high-voltage circuits. Thus the machine may be acting at the same time as a motor and as a transformer, taking energy from certain heavily loaded circuits and transferring it to others more lightly loaded.

CHAPTER VI

VARIABLE SPEED INDUCTION MOTORS

It is in regard to its adaptability to variable speed work, that the induction motor suffers most severely in comparison with the direct-current motor. In fact it may be said at once that the induction motor is distinctly inferior in this respect. For example a shunt-wound, direct-current motor, by the simple addition of a shunt field rheostat, may be made to operate through a considerable range of speed. The efficiency at any of these speeds will be approximately the same, and the speed for any adjustment of the field rheostat will remain practically constant irrespective of the load applied. On the other hand, for certain purposes, such as railway operation, a motor which automatically slows down under load is desired. For this purpose the direct-current series motor is excellently adapted. There is not available on the market at the present time any polyphase induction motor which has either of these two characteristics. It is true that by the addition of a commutator, and the provision of a polyphase regulating transformer, it is possible to build a motor which to a certain extent corresponds to the adjustable speed shunt-wound direct-current motor, but the expense of the necessary additions, together with the complication involved, have so far prevented its commercial application.

The fundamental difference in characteristics is due to the fact that the speed of a direct-current motor depends upon the voltage and voltage relations, while the speed of an induction motor depends primarily upon frequency. The voltage of a circuit is easily changed, but it requires expensive and inefficient apparatus to change the frequency.

The simplest way, and the one most commonly used to obtain speed variation in an induction motor, is to provide means of varying the resistance in the rotor circuit. This is usually done by providing a wound rotor with slip rings, and an adjustable external resistance. The insertion of resistance increases the slip and consequently lowers the speed.

It will be seen at once that this is an inefficient method of speed control. The rotor as was explained on page 32, always requires an

amount of power equal to the synchronous watts, i.e., to the power the rotor would develop if it were operating at synchronous speed with the torque that it is developing. Thus if enough resistance is inserted to reduce the speed to half the synchronous speed, the rotor must receive an amount of power double that which it is developing. The efficiency of the rotor is therefore 50 per cent, and the efficiency of the whole machine is even less than this. If an attempt is made to operate at say 10 per cent of full speed, the efficiency will likewise be less than 10 per cent.

There is also another difficulty. The speed for any setting of the rotor resistance will vary with the load. This is especially true if the

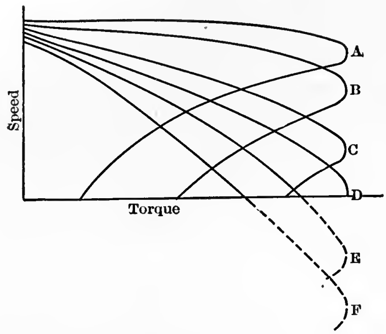


FIG. 40.—Speed Torque Curves of Wound-rotor Induction Motor.

speed reduction at full load is considerable. In Fig. 40 are shown speed-torque curves for various secondary resistances of a typical induction motor. The equation for the torque was derived on page 25 and was found to be $D=\frac{sE^2R}{R^2+L^2s^2\omega^2}$. In this case, E, the applied voltage, L, the inductance of the motor, and ω (equal to 2π times the line frequency) are constant. Assuming then, any value of R, the sum of the internal and external resistance, we can select various values of s, the slip, and solve for the torque D. Instead, however, of plotting the slip, we have used instead the speed $=1-s$. The curve A represents the case where the rotor resistance is a minimum, i.e., the slip rings are short-circuited. In curve $BCDE$, etc., we have constantly increasing resistances. The maximum torque is the same in all cases, but the speed at which it is

developed changes with the resistance. This will be readily seen if we write the above equation in the form, $D=\dfrac{E\left(\dfrac{R}{s}\right)}{\left(\dfrac{R}{s}\right)^2+L^2\omega^2}$. It will be evident that for any slip s, it will be possible to select a value of R such that any desired value of the fraction will be obtained. Hence any possible value of the torque D may be secured at any slip by a suitable choice of the resistance. The dotted parts of curves E and F indicates that to develop the maximum torque, the speed must be negative, that is, the motor must be rotated in the reverse direction. The principles upon which these facts depend have been fully explained, and need not be repeated here.

Control Devices

The wound-rotor machine with slip rings may be employed for either of two purposes; to enable the speed of the machine to be varied, or to give better starting conditions. The methods employed in the two cases are essentially the same, with the obvious difference that in the case of the device intended for starting duty only, much less resistance material is necessary than in the case where the resistance is in circuit constantly. For reasons already given, the rotors of wound-rotor machines are always three-phase. The secondary resistors may be connected either in star or in delta.

A typical motor with its controller and resistors are shown in Fig. 41. This controller in addition to varying the resistance in the rotor circuit is fitted with contacts to open two of the three primary circuits. Thus the current supply to the stator is interrupted when the controller handle is in the neutral position. The contacts are, moreover, so arranged that the connections of the two primary leads to the stator are reversed when the handle is moved from the right to the left of the central position. This has the effect of reversing the direction of rotation of the motor. Moving the handle further from the central position cuts out more of the resistance in the rotor circuit.

In arranging the contacts of the controller circuits it is impracticable to have the secondary resistance balanced at all times. Suppose for example it was desired to have eight running points. If the phases were to be kept balanced at all points, it would be necessary to have seven contacts connected to each of the three resistors or twenty-one in all. By arranging taps as in Fig. 42, the necessary number is reduced to

seven. The various steps are secured by connecting the points 2, 3, 4, etc., successively to the neutral point 1. The resistances may be so arranged that the circuits are balanced on any one of the points

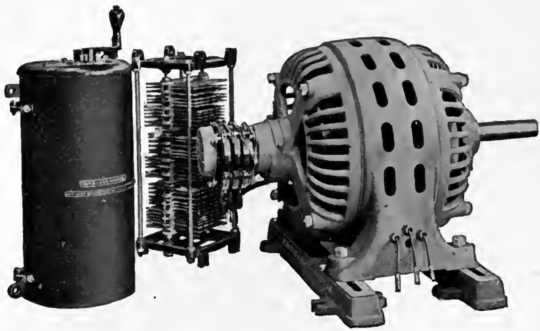


Fig. 41.—Rotor and Induction Motor with Controller. Built by Fairbanks, Morse and Co.

desired. Thus the point of balance may be the first point, or it may be at the point of maximum torque. The unbalancing at the other points has so little effect as to be unobjectionable.

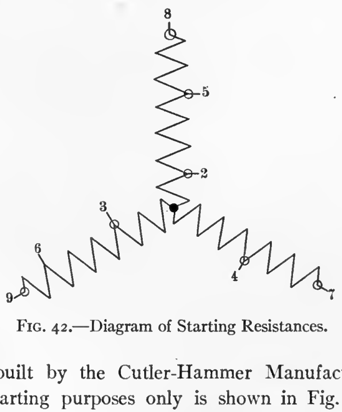


Fig. 42.—Diagram of Starting Resistances.

A rheostat built by the Cutler-Hammer Manufacturing Co. and intended for starting purposes only is shown in Fig. 43. The construction will be obvious from the illustration. In this case also the

starting resistances, are unbalanced. There are only two resistors connected in open V with the three leads connected to the three points of the V. This rather serious unbalancing seems to make but little reduction in the starting torque. Starters of the carbon block type are

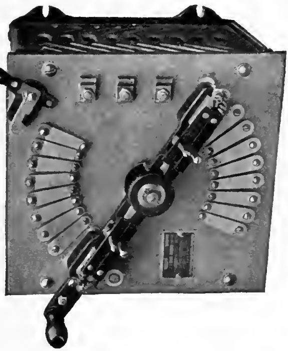


FIG. 43.—Starter for Wound Rotor Induction Motor Built by Cutler-Hammer Mfg. Co.

also built for this service as well as for regulating duty, and these have the advantage of keeping the circuits balanced at all times.

Speed Variation by Cascade Connection

The connection shown in Fig. 44 is known as the cascade connection, or as connection in concatination. Consider for example that both motors in the figure have the same number of turns on the secondary as on the primary, and that both are wound for three phases. Let motor A be operating alone, and let us assume that sufficient resistance has been connected in the rotor circuit so that it is operating at half of the synchronous speed. If the rotor were at rest, the secondary voltage, neglecting the slight drop due to leakage and to the loss in the resistance, would be the same as that of the primary. When operating at half synchronism, the secondary voltage will be just half that of the primary, since the cutting will be half as rapid. Moreover, the frequency of the secondary current will be half that of the primary. It will occur at once to the reader that we might use this current to operate another motor

instead of wasting it in resistance. This can readily be done by connecting the two motors as shown in Fig. 44.

It is necessary, however, that the two motors be connected mechanically as well as electrically. This connection may take the form of a belt and pulleys, it may consist of gears, or the two motors may be mounted directly on the same shaft. The last is obviously the best when conditions permit. If this mechanical connection were not present, the speeds would be the same only when the torques were approximately equal. With lighter torque on the first motor it would speed up and the second one slow down, and vice versa. The sum of the two speeds would under all circumstances be approximately constant.

If, however, the two are mechanically connected as indicated, the set will under all loads run at nearly a constant speed. Assume, for example,

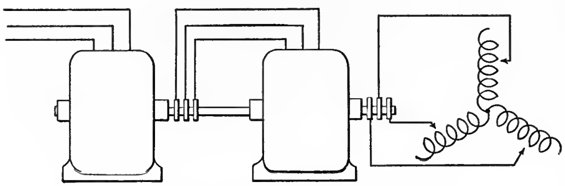


FIG. 44.—Induction Motors in Cascade

that the two motors have the same number of poles, and that they are direct-connected or connected by a one to one gearing. The set will then tend to run at half synchronous speed. That this is so will be readily seen. Suppose for example that the speed is, at a given instant, higher than this. The frequency of the secondary current of the motor A will be less than half the applied frequency, while the frequency of rotation of the second motor will be greater than half of the applied frequency. The machine B will therefore act as an induction generator, and will tend to establish current in the rotor of A against its e.m.f. A will therefore likewise establish current in the supply line, and the whole set will act as an induction generator. The set will therefore slow down until the secondary frequency of A is the same as the frequency corresponding to the rate of revolution of B. The synchronous speed of the set is therefore equal to half the synchronous speed of either one of the motors alone.

The speed of the set when the number of poles of the two motors is

not the same is readily deduced. As previously stated, it is necessary that the speed of the motor B must be such that the frequency corresponding to its speed of rotation is approximately the same as the frequency of the secondary of A. It will be seen that this leads to the rule that the synchronous speed of the set when the two machines are direct-connected will be that corresponding to the speed of a single machine having as many poles as the sum of the poles of the two motors. Thus if the motors have respectively six and ten poles, the individual synchronous speeds on a 60-cycle circuit will be 1200 and 720 rev. per min. The speed of the combined set will be that of a 16-pole machine or 450 rev. per min. With such a set we therefore have available three speeds, and since these are obtained without the use of resistors, the set is working at high efficiency at all of them.

Considering again the connection shown in Fig. 44, let the two motors be exactly alike, and let the rotors have the same number of turns as the stators. They may then be operated singly from the primary supply, or they may both be supplied at the primary voltage and frequency, thus giving twice the capacity of one motor, or they may be operated in cascade at half the former speed. As was pointed out, when operating in cascade, the machine B will be supplied with current at half voltage and half frequency. This voltage is, moreover, correct for this frequency. That this is so will be apparent if we consider that since the voltage and the frequency are both half of normal, the flux in the stator will be of the normal value. This is as it should be, since if the flux were less, we should not get the maximum output from the motor, and if it were much greater we should be in danger of heating or of undue losses.

To understand the action of the set when operating in cascade, let us consider the action of the motor B when operating at normal voltage and frequency and again when operating at half voltage and half frequency. Fig. 45 shows the respective circle diagrams under these two conditions. At will be seen, the current circles are almost the same. That the magnetizing current will be the same is apparent, since the flux remains the same, and the magnetizing current is inedpendent of the voltage or of the frequency (see page 106). The locked current for zero motor resistance will also be unchanged since this current is equal to $\frac{E}{2\pi f(L_A+L_s)}$. The e.m.f. acting on the rotor is evidently half at the lower voltage and frequency, but the frequency being half also and the other quantities remaining unchanged, the current will be the same. The only difference in the circle comes from the fact that the iron losses are approx-

imately half as great at the lower frequency and consequently the circle is somewhat lowered.

While it is true that the circle is practically unchanged, the applied e.m.f. is only half that of the line, and consequently the outupt of the motor is only half as great as its output when operating at primary voltage and frequency. This might have been anticipated from the fact that the motor is operating at half speed. Moreover, since the motor A is likewise operating at half speed, and since on account of the equality of the turns on the primary and secondary, its primary current is practically equal to that of B, its output will also be equal to half its output at full speed. The combined output of the set is then equal to the output of one machine at full speed, or is equal to half that of the two machines at full speed. From this it will be readily seen that the torque of each motor

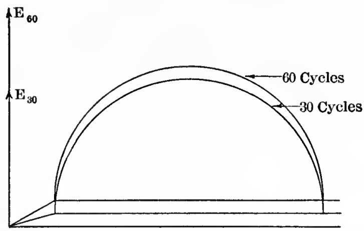


FIG. 45.—Circle Diagram of 60-Cycle and 30-Cycle Motors.

is the same at the low speed as at the high speed. The action, both as regards the output at the half speed, and as regards the torque available, is similar to that of two direct-current shunt motors operated with the armatures in series and in parallel, the fields having at all times full voltage applied.

POWER-FACTOR

It is unfortunately true that the power-factor of two motors operated in cascade is lower than that of either of the motors alone. That this is so is evident when it is remembered that both motors are operating with full flux in the stators. They will therefore require the same volt-amperes excitation as though each were operating alone. The total input to the set is, however, only half of that which it would take if both machines were directly connected to the line. The percentage of watt-

less current is therefore twice as great when the machines are operating in cascade, and the power-factor is correspondingly reduced.

It should also be noted that in the above, it was assumed that the motor B was supplied with current at half the primary voltage and frequency. As a matter of fact, both the voltage and the frequency fall below these values. The output of the second motor is therefore somewhat less than the above considerations would tend to show.

The torque at starting is of importance. As has been shown, the starting torque in synchronous watts is equal to the loss in the rotor circuit. When the two machines are connected in cascade and the set is at rest, the first machine acts as a transformer to supply current to the second machine. The second machine will receive power at the full voltage and frequency, assuming that the first machine acts as a perfect transformer. It will therefore develop its full torque in the same manner as though it were connected to the line. Since, however, there is some drop in voltage the torque will be somewhat reduced.

At the same time the first machine is developing torque. This torque in synchronous watts will be equal as before to the power developed in its rotor circuit. This power is the same as the power in the rotor circuit of the second machine, plus the losses in the rotor of A and in the stator of B. The torque of the machine A is therefore somewhat greater than that of B. The torque in foot-pounds of the machines in cascade is therefore approximately twice that of one of the set, or it is equal to that which would be developed by both of them, operated directly on the primary circuit. The power taken from the circuit if they were operated in this way would, however, be twice as great as with the cascade connection.

Variable Number of Poles

A motor may be constructed with two distinct stator windings, each wound for a different number of poles. If the rotor is of the squirrel-cage variety, the motor will operate at either of the two speeds corresponding to the two sets of poles. If a wound rotor is used, it is necessary that two windings be employed on it also, since a ten-pole stator, for example, would produce no current in a six-pole rotor. The squirrel-cage rotor has no definite number of poles, but automatically adjusts itself to the number of stator poles.

Such a motor will have different outputs at the two speeds. These outputs will be proportional respectively to the two speeds. In this it is similar to the two motors operated in cascade. The principal difficulty

in constructing such motors is the necessity of using very large slots to contain the two windings. This in turn necessitates a greater outside diameter of the motor, and on account of one winding being far from the rotor surface, causes the motor to have a large leakage factor, and a low power-factor. It also reduces the pull-out point, and to a certain extent, the efficiency.

An interesting variation of this plan is to wind the motor with one set of coils which are of rather short pitch for the smaller number of poles. These coils are not permanently connected together, but are so arranged that by means of a special switch the coils may be so connected as to change the number of poles.

The objection to this scheme is the fact that the pitch is not in general the best for either number of poles, and what is of greater importance, the switch is necessarily very complicated. It will be readily seen that with such a switch a short or an open circuit might readily occur. The former would probably lead to burning out a coil; the latter would cause the motor to operate single phase. It is, moreover, apparent that the two windings will not be adapted to the same voltage. If the flux density is kept the same in each, and if the number of coils in series is the same in each, the voltage will be proportional to the respective speeds. Of course, in certain cases, this may be taken care of in the switching device. For example, if the speeds are two to one, twice as many coils may be connected in series for the lower speed. The disadvantages of both of these schemes are such that they are rarely used.

Besides the method of obtaining two different numbers of poles, by utilizing two separate windings, various connections have been proposed by means of which it is possible to change the points of connection of the circuit to a single winding so as to produce the same effect. These are applicable only in cases where the number of poles with one connection is twice that with the other.

In the practical application of this method, the coils are of such a span as to give full pitch with the larger number of poles, and consequently the pitch is half of full pitch when the smaller number of poles is used. The connections are so made that when the motor is operating with the greater number of poles, half of the poles are consequent poles, and the windings as before noted are of full pitch. To obtain the smaller number of poles, it is necessary to conduct the current to what was the center of a winding. To preserve approximately the same flux density at the higher speed, it is necessary at the same time to change the connections of the coils from mesh to star, or for the larger number of

poles the connection is series mesh, and for the smaller, parallel star. The external connections by which this is done are shown in Fig. 46. A full connection diagram for this method of changing poles will be found in " Electric Motors " by H. M. Hobart, page 571 2nd edition.

If the machine is one with a wound rotor, it is of course necessary in

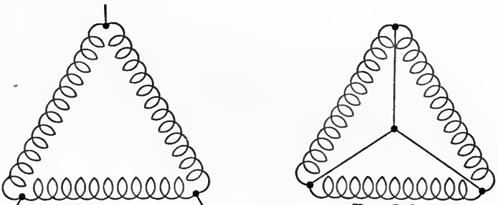


FIG. 46.—Connection for Changing the Number of Poles.

applying any of the methods in which the number of pole is changed, to alter the number of rotor poles at the same time that those in the stator are changed. This involves the use of five collector rings if two separate windings are used, or of six rings if the winding just described is employed. If a squirrel-cage winding is used, it of course adapts itself to any number of poles.

THE COMMUTATOR TYPE POLYPHASE INDUCTION MOTOR

The induction motor, as usually constructed, especially if of the squirrel-cage type, is almost ideally simple. In this respect it is probably superior to any other device for the conversion of any form of energy into mechanical work. As has been pointed out, however, it is inferior to the direct-current motor in several respects, the most important of which are the fact that the speed is not readily changed in an efficient manner, and the fact that the power-factor is always materially less than unity. Both of these objections can be overcome, if we are willing to sacrifice some of the simplicity of the usual motor.

Fig. 47 is a representation of a direct-current armature. The winding shown is a simplex lap. Any other of the direct-current windings might have been used. Imagine this armature placed in an ordinary induction motor stator, instead of the usual rotor. If the commutator were short circuited, by winding a wire around it or otherwise, the

machine would operate in essentially the same manner as a squirrel-cage motor. The currents in the rotor would be as shown in Fig. 7.

Suppose that instead of short-circuiting the rotor commutator in this manner, we make permanent connection to two of the commutator bars as A and B and pass in a direct-current. This would of course have to be done by means of slip rings and brushes. Under these conditions the machine would become a synchronous motor.

The path of the currents in the rotor would be as shown by the arrows. It will be seen that this gives rise to distinct bands of conductors, each band carrying current in only one direction. These currents act to cause poles in the rotor in the positions shown, the observer being supposed to be looking out from the inside of the rotor. As was previously explained, it is perhaps better from the theoretical standpoint to con-

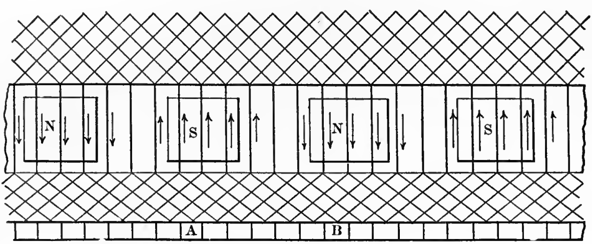


FIG. 47.—Currents in Armature of Direct Current Motor.

sider the reaction of the rotor currents and the stator magnetism, but the concept of poles is very useful in gaining a clear physical idea of the actions involved. It will be seen at once that the " poles " of the stator will attract those of the rotor and carry them around in synchronism with the flux. The machine has thus become a synchronous motor. In a similar manner, a motor with wound rotor may be converted into a synchronous motor by passing direct current in through one of the slip rings and out through one of the others. The third ring would not be used at all, and the corresponding section of the winding would carry no current.

Fig. 48 represents the circle diagram for a synchronous motor. The diagram is not strictly correct, as the effect of the stator resistance has been neglected, but it is sufficiently close for our purpose. The circle marked I may be taken as the circle diagram of an induction motor.

It will be recalled that the reason for the wattless component of the stator current was that this current was required to force the flux across the air-gap. We are now, however, passing direct current into the rotor, and if just enough is supplied to force the required flux through the air-gap, this magnetizing component of the stator current will disappear. The current curve for the circle diagram will then be represented by the circle I_1. The current is now nearly in phase with the e.m.f. for light loads, or the power-factor is nearly unity. A further increase in the direct current in the rotor gives us the curve I_2 or I_3. With such an excitation, the current is leading for most loads, and lagging for very heavy loads only. For any value of the power required, we can pick out such a value of the field current as to make the power-factor unity.

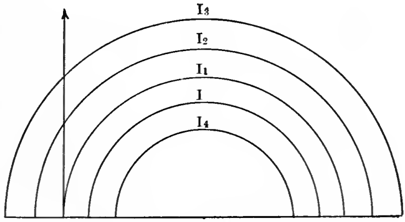


FIG. 48.—Circle Diagrams of Synchronous Motor.

Considering again the direct-current armature, instead of using slip rings as assumed, let us place on the commutator three brushes for each pair of poles. If the armature were wave wound, of course only three brushes in all would have to be used. These brushes should be spaced 120 electrical degrees apart. If three-phase current be passed into these brushes, a rotating magnetic field will be set up in precisely the same manner as would be the case if the current were passed into the stator. Of course two-phase current could be used with four brushes per pair of poles.

This magnetic field will rotate in space with synchronous velocity, and this velocity *will remain the same irrespective of the speed of the rotor itself.* Imagine now that three-phase current is applied to the stator and at the same time to the rotor through the brushes. The connections are supposed to be so made that the fields rotate in the same direction. The relative position of the poles of the two fields will depend upon the

relative position of the taps into the a stator winding and the position of the brushes. If the latter are so set that the two fields are not in the same line, there will be a torque between the fields, and provided the load is not too great, the rotor will start from rest and attain a certain speed. The direction of rotation may be the same as the direction of rotation of the two fields, or the reverse. This latter direction would, however, not be used in practice.

The current in the rotor may be supplied from the line as indicated above, or it may be induced in the rotor, the brushes being either short circuited or connected together by resistances. In this case the motor will operate below synchronism, and the action will be essentially the same as that in the ordinary induction motor, the speed being near synchronism if the resistance used is small, and at a lesser speed as the resistance is increased. If, on the other hand, the current be introduced from the line, the speed will be greater or less than synchronism, as the voltage is applied so as to help or oppose the current which is produced by generated e.m.f. Thus assume the motor to be operating with no load and the brushes short-circuited. The speed will be very near synchronism. Imagine a small e.m.f. applied from the line so as to increase the current through the armature. The increased current will cause an increased torque between the rotor and the stator and the speed of the rotor will increase. This increase of speed will cause the e.m.f. generated in the rotor to decrease, and if the speed is increased above synchronism, to reverse. The generated e.m.f. will then be in opposition to the applied e.m.f. and will tend to cut down the current in the rotor. This will continue until the difference of the two e.m.fs. is just sufficient to maintain enough current to supply the torque required to maintain the rotation.

If, on the other hand, the applied e.m.f. is in such a direction as to oppose the secondary current, this current will be lessened or reversed and the rotor will slow down until the current again reaches the proper value to maintain the rotation. Thus theoretically, at least, we could attain any speed from zero up to the limit imposed by centrifugal force. As a matter of fact, as will be explained later, it is not practicable to utilize all of this speed range.

It will be noted that to attain the above speed variation, use is made of various voltages applied to the armature. These can be obtained by the use of an auto-transformer of the required number of phases or by the use of taps brought out from the stator winding itself. Some difficulty would be experienced in this latter case, since the winding must act as an auto-transformer winding at the same time that it acts as

the primary of the motor. Since no resistance is used the method would be efficient, and the only added losses would be those due to the auto-transformer used.

We have still to explain how the power-factor of the motor may be raised to unity, or the motor even caused to take a leading current. To do this, we may add three more brushes, 90 electrical degrees on the commutator from the main or power brushes. To understand the action, assume the rotor to be caused to rotate at synchronous speed by outside power. Imagine the brushes to be on the commutator as before, but shifted half a pole pitch, so that the rotating field produced in the rotor coincides with that in the stator instead of differing from it by 90 degrees. It is evident that since the two fields are in the same line, there will be no torque between them. If the fields act to magnetize the structure in the same direction, the magnetizing current will be divided between the two in a ratio depending upon the relative values of the e.m.fs. applied to them. Thus if no e.m.f. be impressed on the rotor, the stator will carry the magnetizing current as in the ordinary induction motor. If, on the other hand, no e.m.f. be applied to the stator winding, the rotor will carry all the magnetizing current. Even though an e.m.f. be applied to the stator, the rotor may still carry all the magnetizing current, provided the voltage applied to it is proportionately greater than that applied to the stator.

The question will occur to the reader, What difference does it make whether the magnetizing current is carried by the stator or the rotor? It would make no difference if the rotor were at rest, but when it is operating at synchronism, it can be readily shown that the rotor inductance is lower than when it is at rest, and hence it takes a current more nearly in phase with the applied e.m.f. and consequently at nearly unity power-factor. That the inductance of the rotor at synchronism will be small is readily apparent if we consider that since the rotation of the coils is synchronous with the changes in value of the current, the current in any given coil will never entirely reverse. It is true that the current *varies* somewhat in value in each coil, and consequently there will be some inductance, but it will be less than would be the case if the coils were at rest. We may look at the matter in a slightly different way if we consider that the commutator acts to change the current supplied into a direct current. The action of the machine is then essentailly that of a synchronous motor.

It will be evident that the inductance of the rotor will be practically zero, only when the motor is operating at synchronism. At any other

speed it will be necessary to apply more e.m.f. to the brushes to maintain the required current through the rotor. The compensating action will therefore be correct for only one speed, unless adjusted for each change

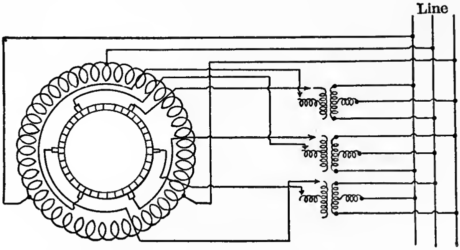


FIG. 49.—Connections of Variable Speed Induction Motor.

of speed. This could be accomplished by interconnecting the auto-transformers supplying current to the power brushes and to the compensating brushes in such a manner that as one voltage was varied, the other would be varied also in the proper manner. As can be readily seen, this leads to considerable complication. The complete connections for a variable speed compensated motor are shown in Fig. 49.

If it is not essential that the motor be adapted for variable speed work, the construction can be simplified by omitting the power brushes and connecting each commutator bar to its neighbor by a small resistor. If the compensating brushes were not applied, the motor would then operate as a plain squirrel-cage induction motor. By adding the brushes the motor can be compensated as before. It is evident that considerable of the current passing into the compensating brushes will "leak" through the resistors connecting the commutator bars. It is therefore essential that these resistors have sufficient resistance, or an excessive loss in

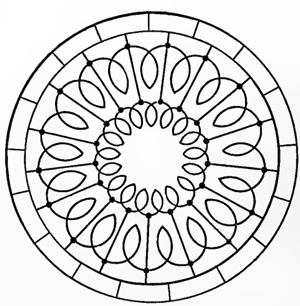


FIG. 50.—Armature of Heyland Induction Motor.

these end connections will result. An armature of this type is shown in Fig. 50.

The action of the commutator in correcting the current of line frequency into a current equivalent to one of the frequency of slip, is very interesting, and in view of the fact that the various authors who have written on this subject do not agree regarding the effect of this action, it is perhaps worthy of further study.

In the first place, it is self-evident that all of the coils between two brushes have the same current passing through them. The current in the section between any two brushes will, however, in general be different from that in an adjacent section. Moreover, since the currents in two adjacent sections unite to form the current in the brush, these currents

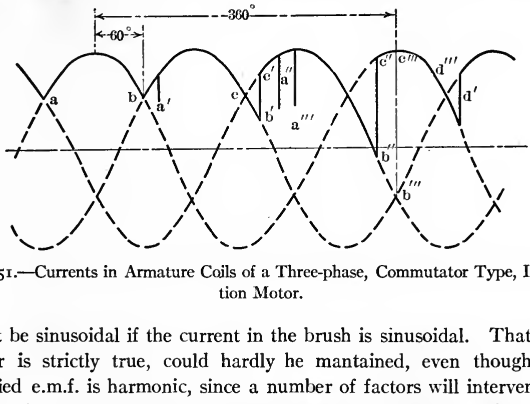


Fig. 51.—Currents in Armature Coils of a Three-phase, Commutator Type, Induction Motor.

must be sinusoidal if the current in the brush is sinusoidal. That this latter is strictly true, could hardly he mantained, even though the applied e.m.f. is harmonic, since a number of factors will intervene to distort the current wave to a greater or less extent. In general, however, we may take this current as sinusoidal without serious error.

In Fig. 51 we have drawn three sinusoids, differing in phase from one another by 120 degrees. These may be taken as being the three currents in three adjacent sections of a rotor fed with three-phase current by means of three brushes spaced 120 electrical degrees apart. If the rotor is at rest, these curves represent the currents in the individual coils of the rotor. If the rotor is in motion, they no longer represent the currents in *any particular coil,* but they still show the current in the *section of the rotor* between the brushes corresponding to the particular curve.

The current in each coil is determined by the fact that as long as the coil in question is between two brushes, the current in it will be sinusoidal.

As the coil passes under the brush and is transferred to the series of coils on the other side, the value of the current in it will be suddenly changed to that value of current existing in the other series. A coil which happens to come to the brush at the time when the current is the same in the two series of coils, will of course undergo no change at the time. This will be true of a coil arriving at a brush at the time when the current has the value " a " in Fig. 51. The curve of current for this coil will be shown by the line *abc*. A coil located 20 electrical degrees from the one first considered would reach the brush at the time when the current had the value a'. The current in it would assume successively the values represented by the curve from a' to b'. As it passed from under the brush, the current would change suddenly from the value b' to c'. The same cycle would then be repeated.

Likewise, a coil located 20 electrical degrees from the second coil would have current in it passing through the values represented by the portion of the curve marked a'' b'' c''. The greatest variation of current while the coil was under the brush would occur in the coil situated 60 electrical degrees from the first coil considered. The curve of current in this case is given by a''' b''' c'''.

If instead of using three brushes on the commutator and supplying three-phase current, four brushes had been used with two-phase current, the variation of current in the coils would have been less. This apparent anomaly is explained by the fact that in this case we should have really had in the armature a four-phase current. In the limit, with an infinite number of brushes supplied with current of a corresponding number of phases we should have no change in current, as the coil passes under the brush, or in other words, each coil would carry a direct current of constant strength.

The magnitude of the e.m.f. that must be applied to the brushes to establish the current through the winding, depends upon the shape of the magnetic field set up by these currents. If this latter were harmonic in space variation and rotated at constant speed there would obviously be no e.m.f. at the rotor brushes at synchronous speed, due to the cutting of this flux by the conductors. This is the case when we have an infinite number of brushes supplied from an infinite number of phases.

In the ordinary case of three or four phases, we do not have this condition. Instead of each conductor carrying a current proportional to the sine of the electrical angle of the conductor from a fixed point of reference, we have a band of current whose width is 120 or 90 electrical degrees, and the strength of the current is the same in all of the conduc-

tors of the band. If as is sometimes the case, the direct-current winding is placed on the armature in addition to a squirrel-cage winding, or if the commutator bars are connected together by resistors as shown in Fig. 50, there will be a powerful action due to the squirrel-cage winding, tending to cause the flux to become harmonic. This was fully explained in connection with the elementary action of the induction motor.

Even though such a winding is not present, there will be a tendency to such action due to the stator winding. If the flux is harmonic and rotates uniformly, the action upon the stator conductors will be to set up a harmonic e.m.f. at the terminals. This e.m.f. is nearly equal and opposite to the applied harmonic e.m.f. Any other variation of the flux would, however, set up harmonics of e.m.f. of higher frequency in the stator winding. These higher harmonics would be short circuited through the resistance and impedance of the stator winding line and the generator supplying the current. The resistance and impedance of these would be small, particularly in case the motor were operated from a generator of large size and through a short transmission line. These short-circuited harmonics would therefore set up currents, which currents would be in such a direction as to tend to destroy the flux which caused them. It is apparent, then, that even in this case, there would be a considerable tendency to force the flux to become harmonic. This action is similar to, but somewhat weaker than, the action of a phase-wound rotor upon the ordinary induction motor.

The above refers of course to the main flux of the motor, i.e., that through both the stator and the rotor. In addition, there is a leakage flux, and the reactance due to this would be approximately the same whether the rotor were at rest or in motion.

At any other than synchronous speed, the curve of current in any conductor would change through the various shapes shown. In this case, since the flux and the coils are traveling at different speeds, there will be an e.m.f. due to the coils cutting through the flux. This will be in such a direction that the applied e.m.f. must be increased for speeds below synchronism, and decreased and finally reversed for those above. From this it will be apparent, as was previously pointed out, that in order to secure power-factor compensation for an adjustable speed motor of this type, it would be necessary to vary the e.m.f. applied to the compensating brushes at the same time that the speed was varied.

It will also be apparent that for this particular type of motor, a two-phase current supply to the rotor would have some advantage over a three-phase supply. It might be that in certain cases this would be of

sufficient importance to justify the transformation of a three-phase supply to the stator, into a two-phase current for the rotor. This could, of course, be readily done by means of Scott transformers or auto-transformers. Even better results would be secured by using six sets of brushes per pair of poles, and operating the rotor six phase.

It will also be apparent that besides the method explained of applying the power current and the compensating currents to separate sets of brushes, the same effect might be produced by applying current to one set of brushes, situated in an intermediate position. The rotating band of current set up in the rotor may be considered as resolved into two bands rotating in the same direction. One of these would lie in such a position as to coincide with the rotating band of flux, and consequently would be in position to produce the torque of the motor. The other would be situated 90 electrical degrees from the first, and would produce no torque, but would be in the proper position to supply the magnetomotive force necessary to force the flux across the gap. This latter component would be nearly constant, and consequently it would be necessary to shift the brushes whenever the strength of the rotor curernt was changed. The same result may obviously be obtained by varying the phase of the e.m.f. supplied and keeping the brushes in a fixed position. This variation in the phase of the applied e.m.f. could be accomplished in various ways. One of the most obvious of these would be to supply each of the phases of rotor current by means of two transformers connected in series, and arranged with their e.m.fs. at right angles. By varying or reversing the components of the e.m.f. any phase desired could be supplied.

To determine the effect of the rotation in reducing the reactance of a rotor, a test was made upon a small wave-wound armature. This was supplied with two sets of brushes, located 90 electrical degrees apart The rotor was mounted in the stator of an induction motor. This stator was provided with a single-phase winding. The iron was, however, slotted uniformly all around the circumference, and as the stator winding was not used in the experiment, the fact that it was single-phase made no difference. Two-phase current at a voltage of about 22 volts and a frequency of 45 cycles was supplied to the rotor. The armature was rotated at various speeds, both in the direction of rotation of the rotating magnetic field and in the opposite direction, and the volts and amperes in both phases measured. From these data the impedance per phase at various speeds can be readily computed. The results are plotted in the curves of Fig. 52, the ordinates being values of the impedance, and the

abscissas, the speed of the motor in per cent of synchronism. These two curves are marked "impedance," and the one at the right is plotted to a scale ten times as large as the other, to show the rapid change near the point of synchronous speed. As will be seen, the impedance decreases very rapidly as synchronism is approached, and increases again beyond this point. The negative values of speed correspond to rotation of the motor in opposition to the rotation of the magnetic field.

It will be noted that the point of minimum impedance is attained at a speed of approximately 110 per cent of synchronous speed. To understand this, we must remember that the e.m.f. generated in the conductors of the armature, due to the cutting of the flux, is in a direction to

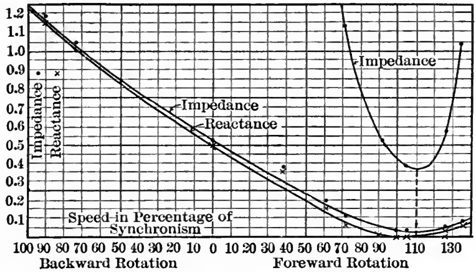


FIG. 52.—Impedance and Reactance Curves.

oppose the applied e.m.f. below synchronism, and in a direction to help it above that speed. It is probable that at synchronism, the reactance is nearly zero and the principal opposition to the current is the ohmic resistance of the winding. Moreover, the e.m.f. required to overcome the reactance is at right angles to that over the resistance. The influence of the reactance is therefore slight at speeds near synchronism, and since above this point, the generated e.m.f. is in the proper direction to help the applied e.m.f., the impedance continues to decrease. Since, however, the reactance increases almost in proportion to the departure of the speed from synchronism, while the total e.m.f., i.e., the sum of the applied and the counter e.m.f. is in proportion to the increase of the speed above zero, the increased reactance soon shows its influence and the impedance again increases.

An attempt was also made to separate the effect of resistance and reactance in the curves. This was rendered difficult on account of the

fact that the resistance of a rotor and brushes is a function not only of the current, being very much less with large currents than with small, but it is also considerably influenced by the speed of the machine. This is on account of the varying contact of the brushes, due to vibration and other causes. The resistance was measured with direct current, the rotor being in motion at about synchronous speed. The results were plotted in the form of a curve, and from this, the resistance drop corresponding to any current could be readily found. This e.m.f. was subtracted vectorially from the total applied e.m.f. The difference divided by the current gave the reactance. These results are plotted in the curve marked "reactance." It of course differs little from the impedance curve for large values of the impedance, but falls very much below it for the points near synchronism. Indeed, judging from the results, the reactance was, as nearly as could be determined, zero, for speeds from about 95 per cent to 105 per cent of synchronism. It was not possible at the time to do the work with sufficient accuracy to determine these values more definitely. In any event it is certain that the reactance at synchronous speed was so nearly zero that it would have been necessary to adopt special precautions to discover its existence.

A similar test was made, the rotor being supplied with single-phase current. Practically no change in the impedance could be discovered as the speed was changed. The constant value of the impedance was 7.3 ohms. That there is no change in this case is due to the fact that with the single-phase current supplied only to the rotor, we did not have a rotating magnetic field. The counter e.m.f. did not therefore decrease in the manner described in connection with the two-phase current. A single-phase motor in operation would in general have nearly a true rotating magnetic field and the apparent impedance would decrease the same as in a polyphase motor.

Commutation

When the rotor is operating at synchronism, the rotor conductors are moving at the same velocity as the stator flux. There is consequently no e.m.f. generated in them and no current produced in the turns short circuited under the brushes. There is, of course, a small reactance voltage due to the sudden reversal of the current in the coil, as in the case of a direct-current machine, but by suitable design this can readily be kept small enough so that there will be no sparking. About the only feature which renders the commutation more difficult than that of a direct-current machine is the fact that, with the same virtual value of direct or

alternating current, the maximum of the wave in the case of the alternating current may have to be commutated, and this is 41 per cent greater than the virtual value. It would therefore be necessary to keep the inductance of the coils somewhat lower than in the case of a direct-current machine.

At any other than synchronous speed, however, the conditions would not be so favorable. Consider, for example, the rotor at standstill. Practically the same flux would cut both the rotor and the stator conductors and consequently there would be generated in each turn of the rotor practically the same e.m.f. as in each turn of the stator. Since we cannot well have less than one turn of the rotor conductor between each pair of commutator bars, we have short circuited at the brushes an e.m.f. equal to the e.m.f. per stator turn. This e.m.f. will set up a large current in the short-circuited rotor conductors. If there were no magnetic leakage, the short-circuit current would assume such a value that the ampere-turns in the short-circuited rotor conductors would be equal to the ampere-turns of all the stator conductors. Since, however, the two windings are on separate cores and since the resistance of the brush contacts and the rotor conductors is considerable, the current does not rise to nearly this value, but would, if not prevented in any way, rise to several times its normal full-load value.

Precisely the same difficulty is met with in the design of single-phase railway motors. The solution of the difficulty is usually accomplished in one of two ways: Either the field flux is kept very weak, so that the generated e.m.f. is small, or else resistance leads connecting the winding to the commutator are used. These, it will be readily seen, are in circuit only with the current from the commutator bars under the brush to the rotor winding, and are not in circuit with the current from coil to coil. The I^2R loss in them is therefore small, and their use does not materially lower the efficiency.

We have considered the commutator-type motor with the rotor and stator connected in parallel to the supply circuit. There is, however, no reason why they should not be connected in series to the line. The motor would then have series characteristics instead of shunt characteristics as in the other case, i.e., it would slow down greatly under load and speed up greatly when the load was removed. It would in fact have essentially the same characteristics as the single-phase, commutator type of motor, except that the torque would be constant during the revolution, instead of intermittent. The motor could probably be built somewhat lighter for the same output, but this advantage would be at least par-

tially offset, especially for railway work, by the necessity of supplying three conductors instead of two. It should also be pointed out that if a motor of this type were used for railway work, it would be possible to arrange the circuits so that it could be used on direct current as well as on alternating. This would of course in general be essential in the case of interurban railroads.

This type of motor has not as yet found extensive practical application. Its possibilities are, however, certainly great, and it is entirely possible that the demand for an adjustable-speed induction motor may bring it into general use. A single-phase motor, operating upon the same general principles and having a compensating winding, is now on the market. This is fully treated in Chapter XIII.

CHAPTER VII

RELATIONS OF FLUX E.M.F. AND CURRENT

We have shown in Chapter I that in the case of an induction motor, particularly if of the squirrel-cage type, when operating at or near synchronous speed, the distribution of flux in space is very nearly sinusoidal, provided the applied e.m.f. is also harmonic. This is, moreover, true irrespective of the type of winding employed, and irrespective of whether or not full-pitch or fractional-pitch coils are used. Various circumstances may modify somewhat this distribution of the flux. As far as the stator current is concerned, its tendency will always be to give a more or less step-like distribution as shown in Figs. 8 and 9. The rotor of a squirrel-cage motor has a powerful damping effect upon any variation of the flux and consequently tends to force the flux wave to retain some given shape. The only distribution which will always give a sine wave of counter e.m.f. is the sine wave, consequently it is the one we should consider in any general discussion.

If the squirrel-cage winding is of high resistance so as to give large starting torque, it is evident that the damping action spoken of will be much weaker, and the wave of flux may depart somewhat from the sine shape. It is also evident that a rotor of the slip-ring type will have a much weaker damping effect than a squirrel-cage rotor. This is true since the corrective currents are not free to follow the exact path required to produce the correction, but are forced to follow through the windings in the order of their connection. These corrective currents themselves will tend to set up variations in the flux and will require currents in the other windings to reduce this variation. The correction is consequently much less complete in the case of a slip-ring motor.

If considerable resistance is inserted in the rotor circuit of a slip-ring machine, the corrective effect is still less, and obviously on open circuit the effect will be zero.

The number of stator slots also has a great influence upon the flux distribution. If the number of slots be made very great, the steps in the

flux curve will be small, and in the limit, with an infinite number of slots, would blend into a smooth curve. To recapitulate, then, we shall have the nearest to a sine distribution of the flux when the number of stator slots is great, the rotor winding is of the squrirel-cage type, with many bars, and is of low resistance. On the other hand, the wave will depart most widely from the sine shape when the stator winding is in few slots, and the rotor is of the slip-ring type and is on open circuit.

Assuming, then, that the wave of flux is distributed in space according to a sine law, the e.m.f. generated in N conductors connected in series, and supposed to be arranged in one slot per phase per pole will be given by $E = 2.22 \Phi N f \div 10^8$, in which Φ is the total flux per pole, and f is the frequency. This is the usual equation of a transformer or alternator. In practice, however, the conductors, except in the case of exceedingly small machines, are never disposed in only one slot per phase per pole. As a consequence the e.m.fs. generated in the various coils of the

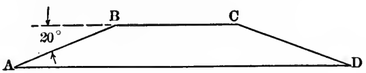


FIG. 53.—Vector Diagram of E.M.Fs.

same winding are not in the same phase, but differ by an angle equal to 360 divided by the number of slots corresponding to two poles. Thus in a three-phase four-pole motor, having 36 slots, the e.m.f. in adjacent coils will differ by 20 degrees.

Such a motor would probably be wound with thirty-six coils, or three coils per phase per pole. The e.m.f. generated by each coil might be computed and laid off as in Fig. 53, the angle between successive vectors being $180 - 20 = 160$ degrees. Thus AB represents the e.m.f. of the first coil, BC that of the second, and CD that of the third. The line AD represents both in magnitude and phase the value of the resultant, and we could readily compute its value by the ordinary methods of trigonometry. If, however, we should adopt this plan, it would be necessary to compute a separate value for each number of slots per pole. It will be readily seen that the different values will not be greatly different, and it is apparent that we can compute one value that will be near enough for all practical purposes. This is especially true, since it is in general unnecessary to compute our values to any very great degree of precision, as many of our assumptions are necessarily not very exact.

The simplest assumption to make is that we have an infinite number

of coils per phase per pole. The vector diagram would then be as represented in Fig. 54. With an infinite number of slots, the e.m.f. due to the coils in each slot will be infinitesimally small compared to the whole e.m.f. and will be represented merely by a dot on the segment of the circle. The resultant e.m.f. is the straight line AB. What we have to obtain is the ratio of the straight line to the segment of the circle. This ratio will give us the ratio in which the e.m.f. is reduced on account of the fact that the coils are distributed, instead of being concentrated in one slot. This ratio for the three-phase motor is evidently

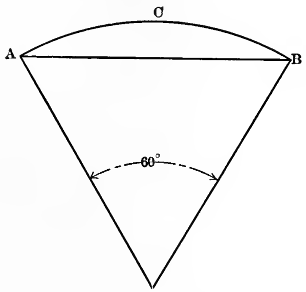


FIG. 54.—E.M.F. Diagram, Infinite Number of Coils.

$$\frac{2 \sin 30^\circ}{\pi/3} = \frac{3}{\pi} = 0.953.$$

In the case of the two-phase winding, the angle spanned by the coils is 90 degrees instead of 60 degrees. Consequently the ratio is

$$\frac{2 \sin 45^\circ}{\pi/2} = \frac{2 \times \sqrt{2}}{\pi} = 0.90.$$

The apparent discrepancy betweed the angle 60 degrees in a three-phase and 90 degrees in a two-phase winding is reconciled, when we consider that all three-phase windings, except in the case of the synchronous converter, are really six-phase windings, reconverted so as to give three phases. The true three-phase connection would give a reduction factor of 0.827. The factor for a six-phase winding is, as shown, 0.953. Hence the six-phase winding will give a larger output, and is therefore preferable.

SHORT-PITCH WINDINGS

Most modern induction motors are wound with short-pitch or fractional-pitch windings, that is, the coils instead of spanning a complete pole pitch or 180 degrees, fall short of this by one or more slots. The

reasons for adopting this type of winding are fully discussed elsewhere. On account of the fractional pitch, the e.m.fs. of the two sides of a coil are not in the same phase and consequently the counter e.m.f. due to a given flux is less, or conversely, the flux to produce a given counter e.m.f. must be greater. The ratio to apply to the whole winding is obviously the

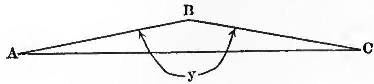


FIG. 55.—Vector Diagram, Fractional Pitch Coils.

same as the ratio for one coil. Thus in Fig. 55 this ratio is equal to the length of the line AC divided by the algebraic sum of the lines AB and BC. If we make AB and BC each equal to one, the ratio becomes $\sin\frac{\gamma}{2}$, where γ is the electrical angle between the two sides of the coil.

There is one other respect in which the induction motor differs from the transformer. The flux along the gap is not distributed in a uniform manner, but follows a sine law. We are usually most concerned with the maximum value of this flux, rather than with its average value. In any sine wave the ratio of the average value to the maximum value is $\frac{2}{\pi}$. If then we designate by $\mathcal{B}_m$ the maximum value of the flux density and by ϕ the total flux, we have,

$$\phi=\frac{2}{\pi}\mathcal{B}_m A.$$

where A is the area of the pole.

Returning now to our transformer equation

$$E=2.22\phi N f\div 10^8,$$

and substituting for ϕ the value obtained above and multiplying by 0.953 in the case of a three-phase winding or by 0.90 in the case of a two-phase, and by $\sin\frac{\gamma}{2}$ to allow for short-pitch windings, we obtain

$$E\text{ (three-phase)}=\frac{1.35}{10^8}A\mathcal{B}_m fN \quad\text{or}\quad E\text{ (two-phase)}=\frac{1.275}{10^8}A\mathcal{B}_m fN.$$

For our purposes the value of $\mathcal{B}_m$ is usually the one to be obtained,

and for this purpose the equations can be more conveniently written in the form

$$\mathcal{B}_m = \frac{7.42 E 10^7}{ANf}$$

for three-phase windings; or,

$$\mathcal{B}_m = \frac{7.84 E 10^7}{ANf}$$

for two-phase windings. If fractional pitch-windings are employed the equations become respectively,

$$\mathcal{B}_m = \frac{7.42 E 10^7}{ANf \sin \gamma/2} \quad \text{and} \quad \mathcal{B}_m = \frac{7.84 E 10^7}{ANf \sin \gamma/2}$$

These equations are the same whether $\mathcal{B}_m$ and A are expressed in centimeter or inch units.

The Magnetizing Current

The determination of the magnetizing current is one of the most difficult problems with which we have to deal. This is due not so much to any complexity in the equations involved, as it is to the uncertainty of the assumptions involved. The problem can be attacked in a variety of ways, and in general the results will differ somewhat. However, the differences are not great, and in no case even approach the uncertainty introduced by the varying length of the air-gap, which is inevitable in practice.

One method of attack is to assume sine waves of *current* in the stator, and no rotor current. The waves of flux will consequently be stepped as shown in Figs. 8 and 9, and will vary from point to point. This assumption may be approximately true in the case of an induction motor with wound rotor, when on open circuit, but it is very far from true in the case of the squirrel-cage machine or in the case of the wound-rotor machine when operating normally with secondary short circuited. The reason for this fact has been pointed out in Chapter I.

The method which on the whole seems the simplest, is to assume that the wave of *flux* is a sine wave. It follows that the magnetizing current will *not* follow a sine law, but we can calculate the value of the equivalent sine wave.

Fig. 56 represents a section of the stator and rotor of an induction motor. The winding is supposed to be full pitch. The small circles represent currents coming toward the observer, the small crosses cur-

rents in the opposite direction. The point of maximum flux will be at the position shown. All the currents at the left, as well as all those at the right, tend to produce flux in an upward direction, i.e., from the rotor to the stator. The student should carefully note that the distance of the conductor from the point A has in itself nothing to do with the tendency which it has to produce flux across the gap. It might appear at first sight that the nearer conductors would exert a greater effect for the same current than those farther away. That this is not so is due to the fact that although the straight parts of the conductors further away do not exert so great an action as do those nearer, the end connections of the conductors further away are longer, and consequently make up for the lesser effect of the straight portions. This is equivalent to the state-

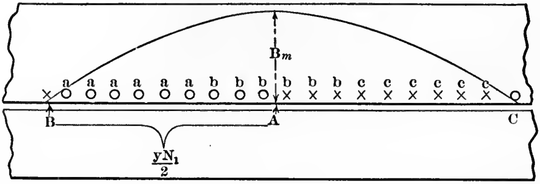


FIG. 56.—Magnetizing Currents of Induction Motor.

ment that the magnetic force at the center of a circular wire carrying a current is independent of the diameter of the circle.

For simplicity let us assume that the number of phases is very great, and that each carries a sinusoidal current. Let the number of phases be y, and let the number of conductors per phase per pole be N_1. Then the number of turns acting on the iron at the center of the pole is $yN_1 \div 2$. Let the maximum current in any conductor be I_m. The average value of the current is obtained by multiplying by $\frac{2}{\pi}$; or,

$$\text{Average current} = I_m \times \frac{2}{\pi}.$$

Also let i_m be the virtual value, that is, the value measured by an ammeter of the current in any conductor, then

$$I_m = \sqrt{2} i_m,$$

or

$$\text{Average current} = I_m \times \frac{2}{\pi} = \frac{\sqrt{2} \cdot 2}{\pi} \cdot i_m.$$

The magnetomotive force is equal to $4\pi \div 10$ times the current and times the number of turns, or

$$\text{m.m.f.} = \frac{4\pi}{10} \cdot \frac{yN_1}{2} \cdot \sqrt{2} i_m \cdot \frac{2}{\pi} = 0.566 N_1 y i_m.$$

The value of the flux per square centimeter is given by the m.m.f. divided by the length of path in the air. This is, making no allowance for the part of the path through the iron. Then

$$\mathcal{B}_m = 0.566 \frac{N_1 y i_m}{d}.$$

Solving this for i_m we get,

$$i_m = 1.768 \frac{\mathcal{B}_m d}{yN_1}$$

or in inch units,

$$i_m = \frac{0.695 \mathcal{B}_m d}{yN_1}.$$

This was developed on the assumption of a very great number of phases. To assume that the same formula will hold for a small number of phases as two or three, is not strictly correct. That this is so is apparent from the fact that with a small number of phases the current in the conductors is not distributed along the gap in accordance with a sine law. The current is in fact distributed in a small number of bands, as shown in Fig. 5. The current in one edge of the band is stronger than we have assumed in developing this theory, and in the other edge it is weaker. As we have previously shown, however, the distribution of the flux remains sinusoidal. In order that this may be so, it is necessary that currents circulate in the rotor bars in such a manner as to offset this tendency of the stator currents to give a stepped distribution of the flux. Thus in Fig. 56 the letters *a*,*b* and *c* represent the conductors belonging to the *a*, the *b*, and the *c* phase respectively of a three-phase machine. It is evident that the current is the same in all of the *a* conductors, and likewise in all of the *b* and the *c* conductors. In Fig. 57 let us consider the instant when the *a* current is zero, and the *B* and *C* currents are equal and opposite. The distribution of the current in the stator is as indicated by the full line marked stator current. The flux, however, on account of the rotor reaction, follows a sine shape as shown. To produce this flux by a stator current alone would require a current distribution as shown by the dotted line. It is evident that there must be enough current in the rotor to compensate for the excess or deficiency of the

stator current. The ordinates of this current are as shown by the dotted line marked rotor current. Fig. 58 shows a similar diagram for the case when the currents in the A and the B phase are equal, and the C current is at its maximum in the opposite direction. The curve

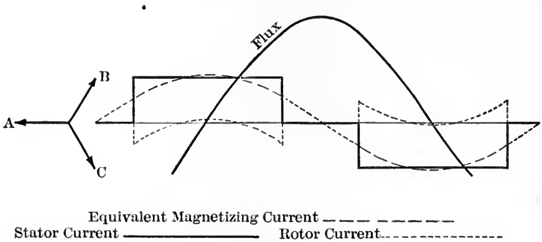


FIG. 57.—Distribution of Magnetizing Currents in Stator and Rotor.

of rotor current distribution is much more distorted in this case than in the first case. Likewise we could draw diagrams for any of the values of the stator current. It will be observed that in each case the rotor current distribution curve contains a fundamental of three times the primary frequency.

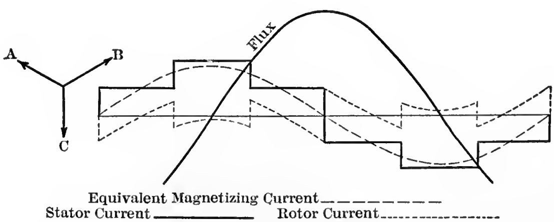


FIG. 58.—Distribution of Magnetizing Currents in Stator and Rotor.

In a similar manner curves could be drawn representing the currents in the rotor bars, when a two-phase stator is used. We can even extend the method to the case of a single-phase stator, and show how the currents in the rotor act to maintain an approximately sinusoidal distribution of the flux, and hence a uniform rotating magnetic field. This case is considered more in detail on page 194, and the curves corresponding to single-phase operation are shown in Figs. 99 and 100.

It must be remembered in considering these curves of rotor and stator currents, that they do not represent the variation with time of the current as is the case with most curves, but on the contrary represent the space variation of the current. The curves as shown are for the condition of no load with the rotor operating as synchronous speed. When the motor is loaded, there is added to the stator current a component in phase with the applied e.m.f. or 90 degrees different in phase from the magnetizing current shown. An equal and opposite current is produced in the rotor, and the total rotor current is then the sum of the added component, and the corrective current shown in Figs. 50 and 51.

It is evident that the corrective current in the rotor causes some copper loss in addition to that due to the stator current. However, the loss due to the magnetizing current is small in comparison with the full-load stator loss, and since the corrective current in the rotor is considerably smaller than the stator magnetizing current, this added copper loss may well be neglected.

It will be evident from what has just been said that the presence of the rotor corrective current will considerably complicate the exact computation of the magnetizing current of a commercial induction motor of two or three phases, and this difficulty will be considerably increased in the case of a motor having short-pitch windings. At least an approximate solution can be obtained in the following manner; In a three-phase motor having say three slots per phase per pole, it will be evident that, with a uniformly revolving sinusoidal flux the e.m.f. in any slot will differ 20 degrees from that generated in a slot on either side of it. The currents, however, in three adjacent slots are the same, since all of these conductors are connected in series. We have shown that for the best operation, each current should differ from its neighbor by an angle corresponding to the angle of displacement of the coils along the core. We may consider then that in this particular case, the current in one of the three slots is in the correct phase while that in each of the adjacent slots differs by 20 degrees from the correct phase. The ampere-turns of each of these should therefore be multiplied by the cosine of 20 degrees. The equivalent magnetomotive force of the three sets of coils corresponding to the three slots, compared to that which we should have if the three currents were in the theoretically correct phase relation is

$$(1+2\cos\theta)\div 3=96 \text{ per cent.}$$

It will be seen that this process of analysis is identical with that employed to obtain the factor by which the e.m.f. is reduced on account

of the fact that the e.m.fs. in the different coils of a winding are not all in the same phase. We may conveniently employ the method used in that case, and consider only the particular instance when there are a very great number of conductors per phase per pole. With this assumption, the breadth coefficient is found to be 0.952 for a three-phase winding and 0.900 for a two-phase winding. It will be seen that the value for the limiting three-phase case does not differ materially from the particular condition chosen of three slots per phase per pole.

The effect of short-pitch winding may be handled in a similar manner. In the ordinary case of a lwo-layer winding, the effect of the short pitch is to shift the one layer through an angle equal to the difference between the angle spanned by the coil and 180 degrees. The magnetomotive force of each layer would then differ by half this angle from the position corresponding to the greatest effect. The correction factor would then be the cosine of half of this angle or the sine of half the angular pitch of the coil. This is again identical with the correction factor found in connection with the calculation of the e.m.f.

It will be seen that the use of each of these factors means that with a given maximum value of the flux density, we must multiply the e.m.f. by a certain expression to correct for the spread of the coils, and by another expression to allow for the effect of short pitch. At the same time, the value of the magnetizing current must be divided by the same factors. The net result is then that the applied voltage is reduced and the magnetizing current increased by the same factor. The value of the total volt-amperes is therefore the same.

This result was pointed out by Dr. A. S. McAllister in his book, "Alternating Current Motors." As is there shown, this result might be expected from the fact that, since the actual energy expended in building up the magnetic field in a particular place (which energy is again restored to the circuit when the field at the point falls to zero) is constant, the apparent watts, i.e., the volt-amperes, should also be constant. Thus in each cubic centimeter of air in which the magnetic density is $\mathcal{B}$ there is stored energy to the amount of $\frac{\mathcal{B}^2}{2.514 \times 10^8}$ joules. If then, the shape of the flux wave remains sinusoidal, the actual energy of the stored magnetic field remains constant, no matter what the type of the winding by which it is produced, and we should expect by analogy that the value of the apparent watts or the wattless volt-amperes would remain constant.

Dividing by the number of phases and by the factors indicated for two- and three-phase, and also dividing by $\sin \frac{\gamma}{2}$ to allow for fractional-pitch winding (the factor becomes one, for full pitch), we obtain as the final expressions for the magnetizing current,

$$i_m = \frac{0.618 \mathcal{B}_m d}{N_1 \sin \frac{\gamma}{2}} \quad \text{(Three-phase, centimeter units)},$$

$$i_m = \frac{0.243 \mathcal{B}_m d}{N_1 \sin \frac{\gamma}{2}} \quad \text{(Three-phase, inch units)},$$

$$i_m = \frac{0.981 \mathcal{B}_m d}{N_1 \sin \frac{\gamma}{2}} \quad \text{(Two-phase, centimeter units)},$$

$$i_m = \frac{0.387 \mathcal{B}_m d}{N_1 \sin \frac{\gamma}{2}} \quad \text{(Two-phase, inch units)}.$$

To test the above theory, use was made of a small squirrel-cage induction motor. This machine was wound with 72 coils in the same number of slots. The coils were connected in six sets of 12 coils each. The circuits were located 30 electrical degrees apart on the core. By various groupings of the coils, the machine could be operated as a single, two-, three- or six-phase machine. To insure the same value of the flux in all of the cases, the applied voltage was varied so as to give 40, 50, 55 and 60 volts over one section of the winding. Thus assuming that the flux was harmonic or at least did not change with the change in connections, the readings were taken for the same value of the flux in each case.

Readings were made as above indicated with the machine connected for one, two, three and six phases. Having adjusted the applied voltage until a certain voltage was indicated across one of the coils, readings were taken of the current in the various phases and of the voltage across the phases. The average current per phase was then multiplied by the average voltage per phase and by the number of phases. The result is the total no-load volt-amperes. To get the true magnetizing current of the motor, it would have been necessary to subtract vectorially from the no-load current the power component of the no-load current. This was not done in this case, as it would have changed the results but little. The resultant volt-amperes with the different connections are plotted in Fig.

59. The ordinates are the volts across one coil, and are therefore proportional to the flux density. Only the points for the single, two and three-phase connection are plotted. The points for the six-phase fall very accurately on the line as drawn, and are omitted to avoid confusion.

It will be seen that the total volt-amperes are practically the same in all of the cases. It is true that the volt-amperes seem a little more in the

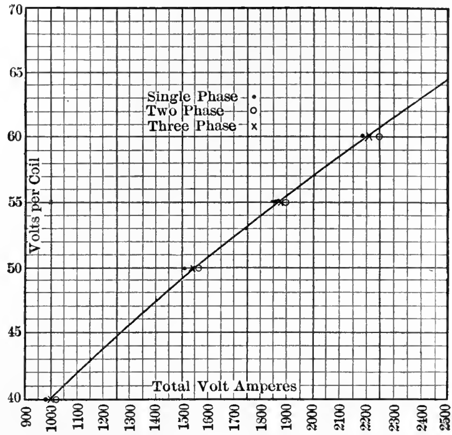


FIG. 59.—Volt Amperes Excitation with Single-, Two- and Three-phase Connections.

case of the two-phase winding, and a little less in the case of the single phase. The difference between the two and three phase is possibly on account of wave shape, although the wave in both cases approximated a sine shape. The connections in the case of the two-phase and the single-phase readings was the same, one of the two circuits being merely opened. Operating single-phase, as will appear later, the flux ceases to be of exactly constant value in the different positions, being somewhat weaker when at right angles to the position of the stator winding. Hence on the whole a slightly lower volt-ampere excitation will be required.

CHAPTER VIII

THE LEAKAGE COEFFICIENT AND THE PREDETERMINATION OF THE CIRCLE DIAGRAM

In Fig. 60 is given a simplified form of the circle diagram. It is obtained from the more general form on the supposition that the iron losses are negligible. This introduces a slight error in our calculations but allows of the derivation of more simple formulæ.

The predetermination of the constants of the circle diagram is of the greatest importance to the designer. In its essentials, it consists in finding the lengths of the two lines OA and OB. The former of these is the magnetizing current of the motor, or it is the current the motor would take if it were operating at synchronism and if it had no losses. Its value is slightly less than that of the no-load current. The determination of the value of the magnetizing current has been treated on page 100.

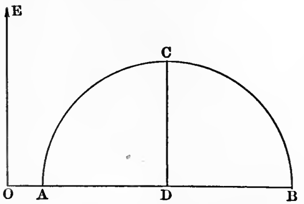


Fig. 60.—Simplified Circle Diagram.

The line OB represents the current the motor would take if it were prevented from rotating and if there were no losses, i.e., if the motor windings had no resistance and the iron loss were zero. In the case of a wound-rotor machine, if we neglect the losses, the current OA is the current taken when the rotor is on open circuit and the motor at rest. OB is the primary current when the secondary is short circuited and is prevented from rotating. AB is the rotor current under the same circumstances on the supposition that we have an equivalent rotor, that is one wound with the same number of turns as the stator.

The determination of the no-load current is comparatively simple, and can be carried out with a considerable degree of accuracy. This is due to the fact that practically all of the flux which is cut by the

stator conductors passes directly across the gap and is cut by the rotor conductors as well. The reluctance of the part of the path which is in the iron is comparatively low, and may frequently be neglected. The path of the flux in the air is quite definite both in length and in area, and almost the only complication arises from the fact that in addition to the current in the stator conductors we have also, even at no load, a current in the rotor conductors.

The estimation of the stator current with locked rotor is by no means so simple. This is due to the fact that a great part of the flux in this case does not follow the path previously indicated. In Fig. 61 is shown a portion of a stator and rotor. The arrows drawn in the slots represent to scale the value of the currents in the respective conductors at a given moment. Since the rotor is prevented from turning there

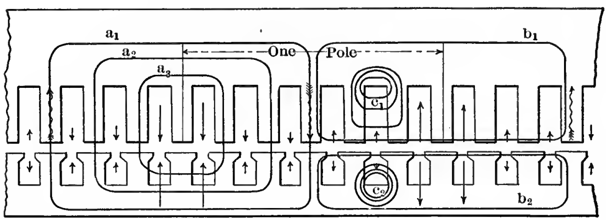


FIG. 61.—Leakage Fluxes of Induction Motor.

will be produced in it a large current. This current will be nearly equal and opposite to that in the stator. The arrows drawn opposite the rotor slots represent to scale the value of the rotor currents. Under these circumstances a portion of the flux will, as before, take the paths indicated by the three lines marked, a_1, a_2, and a_3. This flux is due to the fact that the stator current is somewhat greater than the rotor current and is directly opposite to it in phase. The one will nearly offset the other and the resulting flux is small in proportion to the current.

A little consideration will show that the arrangement of the conductors is substantially that shown in Fig. 62. The crosses indicate currents directed from the observer and the dots currents toward him. Since the current in the upper row of conductors is somewhat greater than that in the lower row, there will be a resultant m.m.f. tending to set up a flux in the paths a_1 and a_2. There will, however, be a much stronger m.m.f. setting up a flux around the paths b_1 and b_2. Also,

since there are spaces between the different conductors of the solenoid, fluxes will circulate as indicated by c_1 and c_2.

In the diagram of the actual motor the paths of the fluxes are indicated by similar letters. As was mentioned, the m.m.f. acting to produce a flux in the horizontal direction is much greater than that acting vertically, but at the same time the reluctance in the path of the former flux is far less than in that of the latter. Hence this flux will not be as large as might be at first thought. It will be greater the narrower the slots are compared with the teeth, the narrower the opening of the slots, and the farther the conductors are from the air gap. Relatively to the flux directly across the gap, it will be greater the greater is the air gap, and the less is the pole pitch.

In addition to the fluxes mentioned, there will also be a leakage flux surrounding the end connections of both the stator and the rotor.

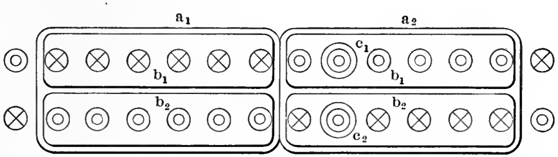


FIG. 62.—Simplified Diagram of Leakage Fluxes.

The part surrounding the rotor conductors will obviously be small in the case of a squirrel-cage machine.

Of these fluxes, those marked by the letter *a* form the useful flux of the motor. The others are leakage fluxes. Of these *c* is known as tooth leakage, *b* as tooth-tip or zig-zag leakage, and that around the end connectors, as end connectors or coil end leakage. Also, since the currents are distributed in bands, and in the case of a wound-rotor machine these bands are continually shifting with respect to one another, there will be set up a pulsating flux surrounding more or less of the conductors of these bands. This is known as belt leakage. This particular leakage thus varies with the relative positions of the stator and rotor. The others are nearly constant. The belt leakage is only about one-third as large in squirrel-cage as in wound-motor machines.

It will be noted that the total flux cutting the conductors of the stator is the same whether the motor is running without load, or is stationary with the rotor locked. This arises from the fact that we

are considering what would happen if the circuits were without resistance. Under this condition, the counter e.m.f. generated in the stator must at all times equal the applied e.m.f. This requires with a constant e.m.f. that the flux be constant.

At no load, this constant flux consists almost entirely of the flux marked *a*, and since it is confined to a definite path, it is comparatively simple to calculate the flux per ampere, or conversely the current needed for a given flux. With locked rotor we have likewise to calculate the flux per ampere, since having this value, we can at once calculate the current required for the total flux, or the length of the line *OB*, Fig. 60. It will at once be apparent that this is a far more difficult task than was the calculation of the no-load current. The student will see the inherent possibility of laying out the paths of the various fluxes, and computing the value of each per ampere, but he will also recognize the enormous difficulty of the task if anything like a general solution is attempted. If a general equation could be derived it would contain among others the following variables: the pole pitch, the length of core, air gap, number and dimensions of slots, permeability of the iron used, type and length of end connections, number of phases, character of the frame, etc. It is evident that to develop or use such a formula would be almost impossible.

In practice, all of the formulas proposed ignore many of these factors. It is tacitly assumed that some of the proportions will be about the average of other motors of similar size, and in case of abnormal proportions in one or more parts, the designer is expected to use his judgment to modify the value obtained. One of the simplest proposals is indicated in Fig. 63. These curves are constructed using as abscissas the values of the pole pitch in inches, and as ordinates the values of the lines of flux per ampere-turn per inch of embedded length of the conductors. For the free length of the conductors we may take the value of one line of flux per inch per ampere-turn, for the case of phase-wound machines, or the value 0.75 for squirrel-cage machines.

To apply this to the case of an actual motor, let us take a machine whose stator has an internal diameter of 17 ins., and a net length of 6 ins. It has 48 stator and 110 rotor slots, and is wound with 48 coils of 11 turns each of No. 6 wire. The machine is wound for three-phase current and is delta connected. It is a four-pole machine and consequently the pole pitch is 13.4 in. From the curve it will be seen that we shall have about 1.7 lines per ampere-turn per inch of core.

This value corresponds to slots three-quarters closed. The calculation is as follows:

Length per turn	42.0 ins.
Embedded length	12.0 "
Free length	30.0 "
Wires per slot	22
Turns per phase on two poles	$11 \times 48 \div 3 \div 2 = 88$

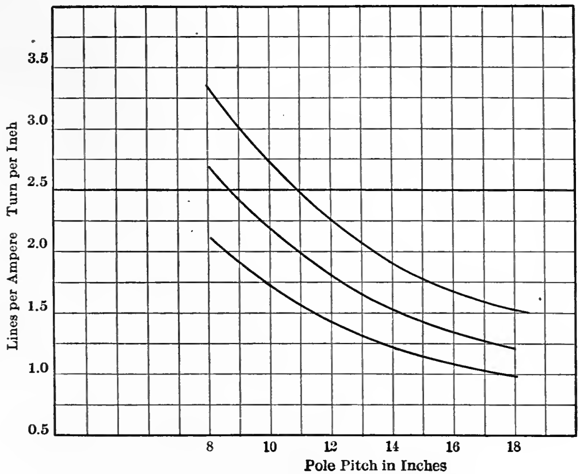


FIG. 63.—Relation of Flux per Ampere Turn per Inch and Pole Pitch.

NOTE.—It is necessary to consider two poles as a unit, since all the flux which passes through the windings of one pole of a pair must also pass through the windings of the other pole of the pair. Consequently, flux generated by the windings of one pole cuts the windings of the other pole of the pair and sets up an e.m.f. in them. Therefore in estimating the inductance of the machine two poles must be regarded as a unit. The e.m.fs. of the various pairs of poles are of course added together.

$$\text{Flux per ampere per pole} = 88 \times 1.7 \times 12 + 88 \times 0.75 \times 30 = 3800,$$

$$L = 2 \times 88 \times 3800 \div 10^8 = 0.0067 \text{ henry}.$$

The inductance of the rotor will be approximately the same as that of the stator, and the total inductance may therefore be taken as being 0.0134 henry.

The machine is wound for 440 volts and the three windings being connected in delta each one is subjected to the full pressure of the line. Moreover, on account of the delta connection, the line current will be equal to $\sqrt{3}$ times the current taken by each phase. Then,

$$\text{Line current} = \frac{440 \times \sqrt{3}}{0{\cdot}0134 \times 2\pi \times 25} = 362 \text{ amperes.}$$

From actual test the locked current of this machine was 376 amperes. The agreement is remarkably good, and is far better than could in general be expected from this method.

Determination of the Leakage Coefficient σ

Most authors, in attacking the problem or predetermining the circle diagram, prefer to determine the ratio of the magnetizing and locked-rotor currents. Then by dividing the no-load current by the value of this ratio, we can determine the value of the locked current. This ratio is usually designated by the letter σ, or $\sigma = OA \div AB$. This procedure has the advantage that the value of σ is a constant of a given frame and slotting, and does not change to any great extent with changes of winding, number of phases, frequency, flux density, etc. It does, however, change for a change in the number of poles. If a method similar to that just given be used, the value of this ratio is at once determined from the ratio of the magnetizing current and the locked current, minus the magnetizing current.

Of all methods proposed for the determination of the value of σ, perhaps that developed by H. M. Hobart is the most general. In brief, the method consists in determining a series of curves, one for each value of H, the average of the rotor and stator slots per pole, and for each value of t, the pole pitch. In all 58 such curves are given. Values of H from 6 to 27 are included, and values of t from 15 to 40. This covers well the range of ordinary practice. In each curve the abscissas denote the gross length of the stator and rotor core. The ordinates are the values of σ. On each curve sheet are drawn three curves corresponding to air gaps of 0.8, 1.5 and 2.5 mm. These curves are stated to apply to motors having open or semi-open slots, and having phase-wound rotors. In the case of squirrel-cage motors, the value

of σ is estimated as being 0.8 of the value for the corresponding phase-wound machine.

It will be seen that this method is perfectly general, and if a sufficient number of curves were determined, could be made to cover any case. To do this completely would require curves taking into account all the possible variations of about fourteen variables. In the curves mentioned, four of these are included. To extend the method to a greater number would require almost a prohibitive number of curves. The method is of course frankly empirical, but should give good results, even taking into account only the four variables. Applying it to the case of the motor previously mentioned we find that σ has the value 0.032. From the actual test we find that $\sigma = 13.6 \div 376 = 0.0361$. Moreover, since the slot opening is small we should be inclined to increase the value found from the curve to somewhat more than 0.032. Hence the agreement in this particular case is good. In numerous other instances the method has been found to give results that are very consistent with the test results.

An extended discussion of the various leakage fluxes has been given by Professor Comfort A. Adams, in Vol. I, Transactions of the International Electrical Congress, St. Louis, 1904, and Vol. XXIV, Transactions of the American Institute of Electrical Engineers. These results have been collected, and the formulæ somewhat simplified by I. E. Hansen, in the *Electrical World*, Vol. XLIX, No. 13. The formulas, while somewhat tedious in practice, are stated to give very good results.

Dr. Hans Behn-Eschenburg has contributed a paper on the same subject to the proceedings of the Institution of Electrical Engineers. He develops the following formula for the estimation of the leakage coefficient.

$$\sigma = \frac{3}{h^2} + \frac{d}{xht} + \frac{6d}{b},$$

in which d is the depth of air gap; h the average number of slots per pole for the stator and rotor; t the pole pitch; b the width of the active iron of the stator and rotor; and x the average width of slot openings. All dimensions are in centimeters.

Applying this to the machine previously discussed we find $\sigma = 0.043$, which is in fair agreement with the value 0.0361 found by test.

Further valuable information in reference to the leakage coefficient is embodied in a number of papers by Rudolph Helmund. Some of

these may be found in the Transaction of the American Institute of Electrical Engineers as follows: Zig-zag Leakage of Induction Motors, Vol. XXVI, p. 1505; Graphical Treatment of the Rotating Field, Vol. XXVII, p. 1375.

Behrend's Method

The following formula for the determination of the leakage coefficient was first proposed by B. A. Behrend in the *Electrical World and Engineer*, November 24, 1900. The formula is simply $\sigma = C\frac{d}{t}$, in which d is the depth of the air gap, and t is the pole pitch. C is a constant for any given set of dimensions, but varies with a change in the design of the motor.

If for σ we substitute its value $\frac{i_0}{i_l}$, in which i_0 is the magnetizing current and i_l is the current with locked rotor, we may write

$$\sigma = \frac{i_0}{i_l} = C\frac{d}{t}.$$

To show that this formula holds, assuming C to be a true constant, it is necessary to prove that i_0 is proportional to the length of the air gap, and inversely proportional to the pole pitch, and that the value of the locked rotor current is not changed by a variation in the values of d and t.

The first of these propositions is almost self-evident, and in any event was fully discussed in deriving the expression for the value of the magnetizing current. That the magnetizing current is inversely proportional to the pole pitch will be apparent if we consider one pole of each of two motors, in one of which the pole pitch is double that of the other. If we assume that the number of conductors is the same in the two cases, it will be evident that the flux density in the air gap of the motor B is half that of A. This follows since the total flux must be the same in the two cases. This being the case, the magnetizing current of B must be half that of A.

To show that a change in either of these two dimensions does not affect the value of the locked current is not quite so simple. Referring to Fig. 61, however, it will be seen that a change in the length of the air gap will have, at most, a minor effect, since the path of any of the

fluxes shown there will be little affected. It is true it has some influence, and in general an increase in the length of the gap tends to increase the locked current and hence to increase the maximum output of the motor. Too great an increase in the air gap would result in all of the flux being leakage flux, and the motor would of course have no output.

That the locked current does not change with a change in the pole pitch is only approximately true. The statement is equivalent to saying that the leakage flux per ampere is unchanged for a change in the pole pitch. Of the three fluxes represented in Fig. 61, b_1 and b_2 will be somewhat decreased, since the path of the lines is somewhat increased, unless we assume that the dimensions of the slots are unchanged, and that all the reluctance is in the gaps between the teeth and in the air gap, in which case the flux would be unchanged. The tooth leakage would be practically the same with the same slots, or it would be lessened if the slots were increased in size to correspond with the increase in the pole pitch. The end connector leakage would not be very materially affected, since the lengthening of the end connectors would tend to increase it and at the same time the lessened bunching of the connectors would decrease it. If, however, too great lengths of pole pitch are used, the end connector leakage becomes of relatively great importance, and makes the design poor.

It will be apparent from the above that the expression as given will be only approximately true, if we regard C as a true constant. In order that the formula should be of universal application it will be necessary to have some means of estimating the value of C. This value is evidently influenced by such factors as the shape of the slots, the type of winding, the length of the core, etc., but as was pointed out, even the factors included in the formula have an influence on it.

In the *Electrical World and Engineer* for April 30, 1904, Mr. H. M. Hobart published curves for determining the value of C. These curves take into account the ratio of pole pitch to core length, and the product of the average slots per pole times the depth of the air gap. Curves are also given both for open and for completely closed slots. Values for partially closed slots can be obtained by interpolation. The curves were derived empirically from the results of tests upon a number of motors.

In "Alternating Current Motors," by A. S. McAllister, it is shown that these curves can be replaced by straight lines, and that values taken from the straight line curves agree equally as well with the facts as do those taken from the original curves. The equations of these lines

are readily obtained, and it is found that the value of C can be represented by the following equation:

$$C=\left(10.5-5.5S_0+2.9\frac{t}{b}\right)\left(0.54+\frac{0.0972}{dh},\right)$$

where S_0 is the percentage opening of slots (average of stator and rotor) t the pole pitch (or in the case of short-pitch windings, coil pitch), d the depth of air gap in inches, and h the average number of stator and rotor slots per phase per pole. If all dimensions are in centimeters, replace 0.0972 by 0.247. In the case of short-pitch windings on account of the overlapping of the phases, the number of slots containing windings of a given phase is greater than would be the case if the winding were of full pitch. It is approximately correct to count as the number of slots per phase per pole, the actual number of slots per pole in which any of the windings of a given phase occur. The fact that some of the slots contain windings of two different phases need not be considered. All dimensions in the above equation are in inches.

Let us consider for a moment the nature of the constant C. We have the equation,

$$\sigma=C\frac{d}{t}=\frac{i_0}{i_l} \quad \text{or} \quad C=\frac{i_0}{i_l}\cdot\frac{t}{d},$$

and we have previously shown that

$$i_0=1.768\frac{B_m d}{yN_1},$$

where y is the number of phases and for exactness should be a large number. Also,

$$i_l=\frac{E}{2\pi fL},$$

where $L=L_p+L_s=$ the local leakage inductance per phase of the primary and secondary. Also we have shown that

$$B_m=\frac{7.07\times 10^7 E}{AN_1pf}.$$

Making these substitutions we get

$$C=\frac{1.768d}{yN_1}\cdot\frac{2\pi fL}{E}\cdot\frac{t}{d}\cdot\frac{7.07\times 10^7 E}{AN_1pf} \quad \text{or} \quad C=7.86\times 10^8\frac{L}{ypbN_1^2}.$$

If now we designate by L_1 the inductance of the motor with locked rotor per pole per phase, per turn, per cm. length of the stator, we get

$$C = 7.86 \times 10^8 \frac{L_1}{y};$$

or, the constant C is the inductance corresponding to one turn of the stator divided by the length of the stator iron and the number of phases and multiplied by 7.86×10^8, or in other words C is equal to 7.86 times the lines of induction set up per turn, per pole, per cm. length, per ampere of the stator with the rotor locked, divided by the number of phases.

That this value will tend to be approximately a constant will be apparent at once. Further, if we let L_2 be the inductance per turn per pole of the stator and consider for the moment that the factor $\left(\frac{0.54+0.0972}{dh}\right)$ has the value one, we can write

$$L_2 = \frac{y}{10^8 \times 7.86}\{(10.5 - 5.5S_0)b + 2.9t\}.$$

From this it appears that the inductance is proportional to a constant times the length of the stator core, the value of this constant depending upon the type of slot used, plus a constant times the length of the pole pitch or a different constant times the length of the end connector, and times the number of phases. We may consider the other portion of the expression $\left(0.54 + \frac{0.0972}{dh}\right)$ as a factor modifying the value of the above-mentioned constants to a certain extent depending upon the length of the air gap and the number of slots per phase per pole. We thus see that there is a definite physical basis for the value of C as derived from the equation. In fact the method in its ultimate analysis reduces to almost the same thing as the method first explained. The principal point of difference is in the modification of the constants for the lines per inch per turn, depending upon the opening of the slots, the air gap, and the number of slots per phase per pole. The application of the method is of course much more simple than that of the first method.

Let us apply this to the case of the machine before mentioned. As before stated, $S = \frac{1}{3}$, $t = 34$ cm. In this case, however, the coil pitch

is from one to ten, whereas full pitch would be from one to thirteen. For the value of t in the formula we therefore use $34 \times \frac{9}{12} = 25.5$. The number of slots per phase per pole in the stator is $48 \div 3 \div 4 = 4$. On account of the short pitch, however, the coils of each phase are displaced sidewise three slots, and we therefore take as the number of slots per phase per pole in the stator the value 4 plus 3 or 7. In the rotor we have $110 \div 12 = 9.18$, giving as the average of both the stator and the rotor 8.09. Substituting these values in the formula we find C to be 8.95. This value of C would indicate a starting current of 386 amperes. The actual starting current, as before noted, was 376. It is therefore in very fair agreement.

Using any of the methods just explained, we can determine all the values necessary to enable us to lay out the circle diagram, and from this diagram we can obtain the value of the maximum input to the motor. It is highly desirable, however, to have a means of determining directly the value of the maximum input, and consequently approximately the maximum output of the motor. This we can readily do by making use of the expression for the value of C which we have just derived.

If E equals the e.m.f. per phase, and there are y phases, we have, letting L equal the inductance per phase,

$$\text{maximum k.w. input} = P = \frac{E}{4\pi f L} \cdot \frac{Ey}{1000} = \frac{E^2 y}{4000\pi f L}.$$

Also we have shown that for large values of y we have

$$C = \frac{7.86 \times 10^8 L}{y p b N_1^2} = \frac{7.86 \times 10^8 p L}{y b N^2}.$$

Substituting this in the expression for the maximum input, we have

$$P = \frac{62500 p E^2}{C b f N^2};$$

or, in inch units,

$$P = \frac{24600 p E^2}{C b f N^2}.$$

As noted, the above expression for C was developed on the assumption that the number of phases, y, was large. In other words, no account was taken of the fact that the e.m.f. generated in the various conductors in the winding of a phase are not in the same phase. To allow for this, we must multiply the sum of all the e.m.fs. generated in

the windings of a phase by the so-called breadth coefficient. This was fully explained on page 97. In this case, then, if we have a three-phase winding we must divide the above constant by 0.953, and for a two-phase winding, by 0.90. This gives us the following values:

$$P \text{ (three phase)} = \frac{25800pE^2}{CbfN^2},$$

$$P \text{ (two phase)} = \frac{27400pE^2}{CbfN^2}.$$

In the case of short-pitch windings, another correction must be made. As was shown, the use of the fractional-pitch winding has the same effect as reducing the number of conductors. To make this correction, we multiply the number of conductors by $\sin \frac{r}{2}$. In the formula for the maximum output, the same factor should be introduced, which gives us as the expression in the case of fractional pitch

$$P \text{ (three phase)} = \frac{25800pE^2}{CbfN^2 \left(\sin \frac{r}{2}\right)^2}.$$

Considering the above formula it will be seen that the output is proportional to the square of the applied e.m.f., and inversely proportional to the square of the number of turns (or conductors) in the complete winding. These relations are obvious in the light of what has already been stated. Nothing else being changed, the input is proportional to the number of poles, and inversely proportional to the frequency and to the width of the motor. The truth of the former will be at once apparent if we consider the motor as a transformer. The maximum current that could be forced through the motor assuming no resistance in the circuits would be inversely as the frequency, the e.m.f., number of turns, etc., being kept constant. Similar reasoning will show that as the width of the motor increases, the inductance will increase and consequently the maximum current will decrease.

The above form of this formula is perhaps the most useful, but by various substitutions we can change it into a variety of expressions which may perhaps help to make clear some of the actions of the

induction motor. Thus in the above, if we substitute for E its value when the number of phases is very great,

$$B_m = \frac{7.07 \times 10^7 E}{ANf}; \quad \text{or,} \quad E = \frac{1.41}{10^8} B_m ANf,$$

we obtain,

$$P = \frac{4.92 pfA^2 B_m^2}{10^{12} Cb}.$$

If we let A_c, the total cylindrical surface of the inside of the stator equal pA, and S_p, the peripheral speed of the rotating flux, equal 10 ft, we obtain:

$$P = \frac{4.92 S_p A_c B_m^2}{10^{13} C};$$

or, the maximum input of the motor is equal to a constant times the peripheral speed of the motor at synchronism, times the area of the inside of the stator, times the square of the maximum value of the flux density in the air gap, divided by the value of the dispersion coefficient. These relations are almost self evident.

If we neglect the loss in the stator conductors, and assume that all of the input appears as torque at the rotor surface, we may equate the maximum input with the work done on the rotor, and letting p denote the pull per square inch of the stator surface, we may write,

$$P = \frac{4.92 S_p A_c B_m^2}{10^{13} C} = \frac{p A_c S_p \times 0.746}{33000}.$$

This readily reduces to

$$p = \frac{2.18}{10^8} \frac{B_m^2}{C}.$$

The interpretation of this equation is that for each value of the constant C, there exists a definite relation between the maximum pull per square inch and the value of the flux density in the air gap. This relation is of course evidently true.

This same relation enables us to obtain a rather curious interpretation of the constant C. The value of p which we have just obtained is the maximum attainable value of the tangential pull per square inch of the stator surface. The value of the radial pull is obtained from the equation:

$$p_r = \frac{B_m^2}{72 \times 10^6},$$

in which p_r is the *radial* pull due to the flux density B_m per square inch. Before comparing these we must remember that p is the average pull all over the surface of the stator. The maximum pull will be greater in the ratio of the square of the maximum value divided by the average value. This ratio, as we have shown, is equal to $\frac{\pi}{2}$. If then p_m is the maximum value of the tangential pull per square inch, we have,

$$p_m = \left(\frac{\pi}{2}\right)^2 p = \frac{5.38}{10^8}\frac{B_m^2}{C}.$$

Letting τ be the ratio of these two quantities, we have

$$\tau = \frac{p_r}{p_m} = \frac{C}{3.88};$$

or,

$$C = 3.88\,\frac{p_r}{p_m}.$$

It thus appears that C may be interpreted as 3.88 times the radial pull per unit area, divided by the maximum tangential pull per unit of area.

CHAPTER IX

Some General Considerations Relating to Design

HIGH- VS. LOW-FREQUENCY MOTORS

A number of the facts previously discussed have reference principally to the design of the induction motor. It is the intention to group in this chapter a number of facts and formulas of interest principally to the designing engineer, rather than to the operating engineer.

The question of whether a high- or a low-frequency current is preferable for operating induction motors is one which can be answered in different ways, depending upon the circumstances. In this country the choice usually lies between 60 and 25 cycles. In the following discussion, however, the frequencies of 50 and 25 cycles will be considered on account of the greater ease of making comparisons. In general we might make the statement that for small motors, taking everything into consideration, the higher frequency is preferable, and on the other hand, for large motors, the lower frequency is the more desirable. Another way of stating the same thing is to say that *for the same speed of rotation*, the low-frequency motor will have the better electrical characteristics. It will, however, usually be somewhat higher in price. We can perhaps best establish the truth of these statements by examining a couple of typical cases.

Consider first the case of a 10-h.p., three-phase, 50-cycle, 440-volt motor. A machine for this frequency, (in all sizes from 10-h.p. down) would usually be wound for four poles, giving a synchronous speed of 1500 rev. per min. This is assuming that the motor is for ordinary belted service. For special applications, many other speeds may be needed. For 25-cycle service the speed adopted for all sizes from 50 h.p. down would in most cases be 750 rev. per min. Of course these motors could be wound with two poles, giving a speed of 1500 rev. per min., but this is rarely done. The objection to doing so is that since the flux per pole is twice as large as in the four-pole type, the section

of iron back of the slots must be twice as great, for the same rotor diameter. Moreover, the end connections are very long and the machine is difficult to wind. On account of these facts, the two-pole machine is rather rarely used except in the very small sizes. This inherently lower speed is the fundamental reason why the low-frequency machine is less satisfactory in the small sizes than the high-frequency one.

To study this more in detail, let us consider the case of the 10-h.p., 50-cycle, 1500-rev. per min., 440-volt machine mentioned above. If it were desired this same machine could be operated without change upon a 25-cycle, 220-volt circuit. The motor might, it is true, be slightly improved by somewhat increasing the flux density, but for our purposes we may assume that no change is made in it. The flux density is obviously the same as before, since the frequency and the voltage are both halved. Hence, the torque will be the same for the same current, and since the speed is only half as great, the rating will now be 5 h.p. The first great disadvantage of the low-frequency machine is at once apparent. The cost of a 5-h.p., 25-cycle machine is practically the same as the cost of a 10 h.p., 50-cycle machine. It is true that if a resistance-type starter is used, something may be saved in the cost of this item, but if an auto-starter is used, the cost of the 5-h.p., 25-cycle starter will be practically the same as the cost of one for the 10-h.p., 50-cycle motor. The fact that the machine operates at a lower speed is in itself a slight advantage, and will lead to more quiet operation and lower maintenance charges. However, giving these facts their full value, it is still true that to equip a factory with 25-cycle motors of small and medium sizes, costs far more than would be the case if they were to operate on 50 cycles.

To deduce the comparative electrical properties, consider the circle diagrams of the two machines. Since the frequency is half as great in the one case as in the other, and since the proper voltage is also approximately half as great, the diameter of the circle will be the same in the two cases. Hence it will be at once apparent that the power-factor, the overload capacity and the starting torque will be the same. The overload capacity and the starting torque are of course supposed to be expressed in percentage of the full-load torque.

To compare the efficiencies, let us assume that in the 50-cycle motor the iron loss is 500 watts, the copper loss 500, and the friction loss 200 watts. In the 25-cycle machine the iron loss, since the density is the same, will be somewhat less than half, say 225 watts, the copper loss will be unchanged, and the friction loss will also be less than half, say

75 watts. The efficiency of the 50-cycle motor is then 86.2 per cent that of the other, 82.2 per cent.

It would be fairer to compare motors of the same horse-power rating, and if this were done the discrepancy in efficiency would not be so great. The power-factor of the low-frequency motor would probably be somewhat higher. The cost, while much greater, would by no means be twice as great. The actual ratio at present prices is about in the proportion of 1 to $\sqrt{2}$ or 1.41.

When we consider motors large enough so that the speed is limited by other factors than the characteristics of the motor, the result is quite different. Consider the case of a 100-h.p. motor. Such a machine would probably be wound to operate at 500-rev. per min. for either frequency. In this case all the advantage except in the point of cost is with the low-frequency motor. That the cost of the 25-cycle machine is higher is due to the fact that since each pole is twice the size of those of the other motor, the section of iron to carry the flux must be greater. It is usually not twice as great, since the density may be greater on account of the lower frequency, but it is materially more. The end connections are likewise about twice as long, and consequently the cost of copper is greater. For much the same reasons the cost of the auto-starter is also greater. The ratio of costs at the present time is about as 1 to 1.1.

In comparing the electrical characteristics of the two machines, we may, for the sake of simplicity, assume them to have rotors of the same diameter and length. The tendency in practice would be to make the 25-cycle machine of a smaller diameter and a greater length than the 50-cycle motor. The comparison is therefore somewhat unfair to the machine of lower frequency. We may likewise assume for the sake of simplicity that the flux per square inch in the air gap is the same in both cases, and that the air gap is the same.

The first noticeable point is that since the ampere-turns needed to maintain the flux across the air gap depend only upon the flux density and the length of the gap, the ampere-turns per pole will be the same in the two cases. Since the low-frequency motor has only half as many poles as the other, the total ampere-turns will be only half as great. Hence the no-load current will be only half of that taken by the 50-cycle motor. As can readily be seen from the circle diagram, the power-factor of the low-frequency motor will be much higher. The fact that the leakage path from pole to pole is twice as great, improves the leakage coefficient and likewise the power-factor of the low-frequency machine.

Since its circle diagram is so much larger, the pull-out point, the starting torque, and the point of maximum power-factor will all be greater.

Whether or not the efficiency will be greater depends upon a number of features. The fact that the frequency is lower will tend to make the iron loss less, but on the other hand the copper loss will be greater on account of the longer end connections. Probably the efficiency will not be greatly different in the two cases. In the above it must be remembered that full credit has not been given to the 25-cycle machine, since we have assumed that it followed the design of the 50-cycle motor. It could be improved in a number of particulars by various modifications in the design. However, in any case it is obviously the better machine of the two from the electrical standpoint.

Open or Closed Slots

In this country the completely closed slot is rarely used. This is on account of the excessive cost of winding such a motor. The wires must be threaded through one at a time from the end, and the process is necessarily tedious and expensive. Abroad, where labor cost is less, such slots are sometimes used; in this country as far as the writer knows, they are never employed. When, therefore, closed slots are referred to, partially closed slots are meant. In these slots an opening is left about twice as wide as the thickest wire to be used. The coils are first wound up on a form, and a short distance at each end is then taped, so as to hold the coil in shape. The slot insulation, consisting of paper, oiled muslin, etc., is placed in position, and the wires of the coil are slipped into the slot, one at a time. The projecting parts are taped after the coil is in place.

In the case of the wide, open slots, on the other hand, the coils are completely formed, taped, in some cases impregnated with insulating material, and are then placed in position. The slot lining in this case is very thin, being merely sufficient to protect the coil from injury while it is being placed in the slot.

Of the two constructions, there is little doubt that from the customer's standpoint, *given two motors of identical characteristics*, the one with open and the other with closed slots, the former would be preferred. This is on account of the fact that with the open slots a better opportunity is presented to insulate the coil, and since in case a coil does break down, it can be replaced somewhat more easily than would be the case with closed slots. The difference in the time of replacement

is not, however, as great as might be thought, since, in either case, a large amount of time is spent in disconnecting and reconnecting the coils, and this time is the same with either open or closed slots.

In many cases it is impossible to build a motor with open slots with as good characteristics as one with partially closed slots. Thus, in Fig. 64, let the full-line curve represent the circle diagram of a motor with closed slots. If, keeping exerything else the same, the slots are made open instead of closed, there will be two changes apparent in the diagram. In the first place, the line OA, representing the magnetizing current, will be lengthened to, say, OA'. This is so since the flux now crosses from stator to rotor in tufts instead of being nearly uniform. The flux density in the tufts is of course greater than would be the case if the iron were unbroken, and the magnetizing current will

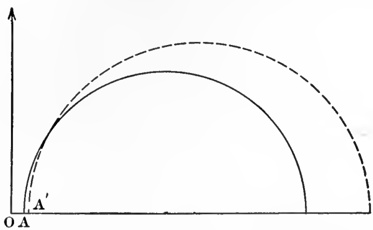


Fig. 64.—Effect of Open Slots upon Circle Diagram.

be correspondingly increased. The other effect is that the diameter of the circle will be greater. This is so, since on account of the open slots the tooth-tip leakage is less, and consequently the impedance of the motor is less. The corresponding circle diagram of the open-slot motor is shown by the dotted lines.

It is evident that in the open-slot motor the power-factor is lower at light loads and higher at heavy loads. The maximum power-factor may be better or worse. If the diameter of the circle is increased in a greater proportion than the magnetizing current, it will be better, and vice versa. If the two are increased in the same proportion, there will be no change in the maximum power-factor. In most cases‘ if the closed-slot motor were designed so that its characteristics were the best that could be secured with the given core, those of the open-slot motor would be poorer. Thus the maximum value of the power-factor will come at too large a load and will be lower throughout the operating range of the motor. On account of the greater current

required, this will result in a lowering of the efficiency of the motor throughout its normal range, and a slight improvement at overloads. It is true that the pull-out point and the starting torque will be higher, but in general the motor will be slightly inferior to the closed-slot motor, so far as its electrical characteristics are concerned, and slightly superior from a mechanical standpoint.

The exception to this is the case of motors of long pole pitch. Such motors are usually but not necessarily low-frequency motors. The ampere-turns required to maintain the flux across the gap are proportional to the number of poles, to the length of the air gap, and to the flux density in the gap, but are entirely independent of the length or width of the poles. The same thing is true of any electrical machine, either direct or alternating. The large poles, other things being equal, require more copper on account of their greater size, but not more ampere-turns. Thus compare two motors wound on the same frame for the same speed and the same output, one having eight poles for 50-cycle current, and the other four poles for 25-cycle current. The two motors will have approximately the same full-load current, but the magnetizing current of the 25-cycle machine will be only half that of the other.

It will be evident that in the case of such machines as, for example, large four-pole, 750-rev. per min., 25-cycle machines, the magnetizing current will be a comparatively small proportion of the full-load current. Hence it may readily happen that opening the slots will so much increase the diameter of the circle as to offset the larger magnetizing current. This is especially so since such motors usually have generous air gaps, and opening the slots has then a comparatively small effect upon the magnetizing current.

The above considerations will also serve to emphasize the difficulty of designing motors for low speeds, small outputs and high frequency, i.e., with narrow poles. If such conditions must be met, the only remedy is to use nearly closed slots, and the shortest possible air gaps. Such machines should be avoided if possible, by employing back-geared motors, belts or other devices.

Best Diameter of Rotor

It is obvious that in designing an induction motor it may be made of large diameter and short along the shaft, or of great length and small diameter. The selection of the most appropriate dimensions is of great importance. Like most other problems in designing, this does not admit of an exact solution, since so many antagonistic factors are

involved. We can, however, point out the considerations which should govern the designer in deciding these questions.

The first point to be kept in mind is that the product of the square of the diameter of the rotor times the axial length of the stator iron is approximately a constant; or

$$bd^2 = K_2.$$

This is based upon the supposition that the flux density and the number of ampere-turns per inch of the periphery are independent of the diameter of the rotor. While it might be advantageous to change these quantities slightly as the diameter of the rotor was changed, yet the assumption is nearly exact. Assuming this to be the case, it will be seen at once that the output of the motor will be directly proportional to the length of the motor. It will also be proportional to the square of the rotor diameter, since if this latter is doubled, the pull per inch of periphery will be unchanged, but the periphery will be doubled, and the speed of the moving conductors will also be doubled. Consequently the output of the machine will be quadrupled, or it will be proportional to the square of the rotor diameter.

Economy of Iron

The amount of iron required for the core is not affected by the diameter of the rotor. Thus in Fig. 65, let 1 represent the core of an induction motor, and let 2 represent the core of a machine of the same rating but of a diameter k times as great. It will be necessary to make the radial depth of iron k times as great, since each pole is k times as long, and the extra depth is necessary in order to carry the greater flux per inch along the shaft. This necessitates making the outside diameter and the diameter of the opening in the rotor k times as great. The length of the iron along the shaft, on the contrary, will need to be only $\frac{1}{k^2}$

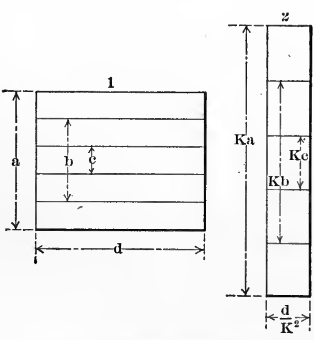


Fig. 65.—Relation Between Weight of Iron and Diameter of Core

times as much. These deductions neglect the fact that the depth required for the teeth does not greatly change with the diameter. It will be readily seen that the volume of iron in 2 is equal to

$$[K^2(a^2-c^2)\frac{\pi}{4}\cdot\frac{d}{K^2}=d(a^2-c^2)\frac{\pi}{4},$$

and it will be apparent at once that the same expression is the volume of iron in 1.

If we were to take account of the fact that the depth required for the teeth does not change, we should find that there is a slight economy of iron by using the greater diameter.

Copper Economy

As far as the *active* copper, i.e., the copper in the slots, is concerned, the greater the diameter the less the amount of copper required. This is so since with a greater rotor diameter, each element of the winding is cutting the same flux as a greater velocity. The current, carrying capacity will be at least as great, and consequently the output per pound of copper will be greater. If, however, we increase the diameter, keeping the number of poles and the speed constant, the pole pitch, and consequently the length of the end connections, is increased. After a certain diameter is reached, the increase in the copper in the end connections will more than offset the saving in the active copper.

It is not difficult to compute the diameter which will give the greatest economy of copper.

Let K denote the weight of copper per inch of periphery and per inch of length along the shaft; p the pole pitch in inches; N the number of poles; b the length of the iron of armature parallel to the shaft; d the mean diameter of winding; r the ratio of length of end connectors to pole pitch, and W the total weight of copper.

Then,

$$W=k\pi d(b+rp),$$

and since as we have previously shown $d^2b=$const. $k_1{}^2$

$$W=k\pi d\left(\frac{k_1}{d^2}+\frac{r\pi d}{N}\right).$$

Then for the minimum value of W,

$$\frac{dW}{d \cdot d} = 0 = \frac{2k\pi^2 rd}{N} - kk_1\pi d^{-2};$$

or,

$$2\pi rd^3 = Nk_1$$

But, since $p = \frac{\pi d}{N}$, substituting for N and k_1 we have

$$b = 2rp.$$

Since b is the active length of one conductor and rp is the inactive length, for minimum copper the shape of the armature should be such that the active wire is twice the inactive. If we assume that the end connector is 1.5 times the coil pitch the rule may be stated thus. The shape of the armature for the minimum amount of copper should be such that the coil pitch is one-third of the active length of the armature or $p = 0.33l$.

The above applies to direct as well as to alternating-current generators and motors.

Diameter for Best Power-Factor

In the above we have shown the diameter of rotor which will require the smallest amount of copper. This diameter will also be the one which will give nearly the highest efficiency or the lowest losses. This will be apparent when we consider that the amount of iron, and consequently the iron loss is independent of the rotor diameter, and since the diameter deduced above requires the smallest amount of copper, it will give the smallest copper loss. The friction and windage loss would be slightly less with a smaller diameter, while with a greater one the power-factor would be better and consequently the efficiency would be slightly higher on account of the slightly smaller current. The last two considerations about offset one another, and hence we may say, the diameter for the minimum amount of copper is also the diameter for the maximum efficiency.

It remains to consider the diameter which will give the best power-factor. In order that the power-factor may be high, it is necessary that the leakage factor be as low as possible. This follows from the fact that the maximum value of the power-factor is given by the expression max. $\cos \theta = \frac{1}{1+2\sigma}$. We have also derived the equation

$$\sigma = C\frac{d}{l},$$

We have also shown that σ, the leakage factor can be represented by the expression

$$\sigma=C\frac{d}{t}=\left(10.5\frac{d}{t}-5.5S_0\frac{d}{t}+2.9\frac{d}{b}\right)\left(0.54+\frac{0.247}{dh}\right).$$

It must be kept in mind that this expression is an empirical one, derived from experience, although the general form might have been predicted from theory. The results of any mathematical transformations will be of value, only as the original formula represents the facts. In the experience of the author as well as in that of others, the formula has been found to accord well with the results obtained from the completed machines.

From the above we have,

$$\sigma=C\frac{d}{t}=\left(10.5\frac{d}{t}-5.5S_0\frac{d}{t}+\frac{2.9d}{b}\right)\left(0.54+\frac{0.247}{dh}\right).$$

Since we wish to study the effect of varying t and b, everything else may be regarded as being constant, and we may then write:

$$\left(0.54+\frac{0.247}{dh}\right)(2.9d)=K.$$

We have previously shown that in an induction-motor design $d^2b=$ const. Since t is proportional to d we may write instead $t^2b=K_2$. Making these substitutions we have

$$\sigma=K\left\{(3.62-1.895S_0)\frac{1}{t}+\frac{t^2}{K_2}\right\}.$$

Then for σ a minimum,

$$\frac{d\sigma}{dt}=0=\frac{2t}{K_2}-(3.62-1.895S_0)\cdot\frac{1}{t^2};$$

or

$$t^3=K_2(1.81-0.95S_0).$$

Substituting the value of K_2 we have:

$$t=(1.81-0.95S_0)b,$$

in which p is the pole pitch or in the case of short-pitch windings the coil pitch, b is the core length, and S_0 is the percentage opening of the slots. Thus we have

For wide-open slots, $t=0.86b$.

For half-open slots , $t=1.33b$.

For closed slots , $t=1.81b$.

It will be noted that to obtain the maximum power-factor, we require a much greater pole pitch, or what is equivalent, a much greater armature diameter, than is required to obtain the minimum amount of copper, and the best efficiency. It should also be noted that, contrary to a rather prevalent belief, an increase in the diameter of the rotor does not always result in an improvement in the power-factor.

Length of Air Gap for Best Power-Factor

Writing as before,

$$\sigma=\left(10.5\frac{d}{t}-5.5S_0\frac{d}{t}+\frac{2.9d}{b}\right)\left(0.54+\frac{0.247}{dh}\right),$$

and changing this to the form

$$\sigma=\left(0.54d+\frac{0.247}{h}\right)\left(10.5\frac{1}{t}-5.5\frac{S_0}{t}+\frac{2.9}{b}\right),$$

we see at once that this will be a minimum for $d=0$, and will have the value

$$\sigma=\frac{0.247}{h}\times\left(\frac{10.5}{t}-5.5\frac{S_0}{t}+\frac{2.9}{b}\right).$$

From this it is apparent, as might have been expected, that the smaller the air gap, the better will be the power-factor of the motor. Of course mechanical considerations will deter us from making the gap as short as would be desirable. The practicable limits will usually lie between the values 0.02 in. and 0.125 in. The larger values are practicable only with motors of large pole pitch. This implies in general motors for low frequency.

Modifications Introduced in Practice

In practice, the application of the above principles is modified by the fact that it is desirable to use as few different diameters as possible· This is of course on account of the reduction thereby made possible in the number of blanking punches, bearing arms, etc. In some cases it is customary to build three different sizes of motors of the same speed on the same diameter of frame. Thus a motor of 10 h.p. at 1200 rev. per min., might have a core of 16 in. diameter and 4 in. long. The 15- and 20-h.p. motors of the same speed might have the same diameter, and net core lengths respectively of 6 and 8 inches. The three motors would have gradually better characteristics as they increased in size. Thus the efficiency would be better in the larger motors, since the percentage of idle copper would be less. The percentage loss in friction and windage would likewise be less in the larger machines.

The leakage factor would also improve as the core length increased, since the reactance of the end connectors would be proportionately less. Consequently the power-factor and pull-out point of the larger motors would be somewhat better.

Size of Copper Conductors

The determination of the size of conductors to use in any electrical machine may be considered from two standpoints. The conductor *must* be large enough to keep the temperature rise within a certain limit. They should be large enough to prevent excessive power loss for the type of machine in question. If the size of the conductor used did not influence any other dimension of the motor, the problem of determining the proper cross-section for given conditions would be comparatively simple. This, of course, rarely happens. The following can be considered as applying only to a case of the kind mentioned. For example, a design might be under consideration in which there was plenty of room in the slots for either of two sizes of wire. The smaller might be satisfactory as far as heating was concerned, and the question would arise whether or not it was advisable to use the larger wire on account of the better efficiency attainable.

As will be shown presently, the loss per pound of copper in watts can be expressed by the formula,

$$P_c=8\times\left(\frac{700}{A_a}\right)^2,$$

where A_a denotes circular mils per ampere.

If we assume that the motor or other apparatus is used an average of six hours per day, 300 days in the year, and that the value of the energy wasted is \$0.01 per k.w.-hr., the cost of the energy lost per year per pound of copper is,

$$6\times300\times0.01\times\frac{8}{1000}\times\left(\frac{700}{A_a}\right)^2=\frac{70,600}{A_a^2}\text{ dollars.}$$

If, on the other hand, the value of the copper installed in the motor be assumed to be \$0.25 per pound, and the interest and depreciation be taken as 10 per cent, we have a fixed charge of \$0.025 per pound of copper per year. The total cost of operation will be approximately a minimum when the interest cost is equal to the value of the power lost, or,

$$\frac{70,600}{A_a^2}=0.025.$$

Solving this we get,

$$A_a=1680.$$

Since this result is based on the supposition that nothing else is affected by the change in the size of the copper conductors, about the only practical application we can make of it is the conclusion that the conductor should, considered by itself, be in general much larger than would be dictated by other considerations. If, therefore, we could in a particular case use a wire one size larger than would ordinarily be employed, without increasing the size of the slots or altering any other dimensions, it would pay to use the larger wire. Of course even this limited use of the deduction would in many cases be prohibited by commercial conditions. It also indicates the most economical size of wire to use in the leads and the coil connections. If a motor is to be used a less number of hour per year than above indicated, a smaller size of wire would be preferable.

Two-Phase and Three-Phase Motors Compared

In every respect, the two-phase motor is slightly inferior to the three-phase. The difference is slight, it is true, but it unquestionably exists. This slight inferiority is mainly due to the fact that in the three-phase motor, the angular breadth of one-phase winding is 60 degrees, while in the two-phase it is 90 degrees. This results in the breadth coefficient in the three-phase motor being better than in the two-phase.

As was shown on page 98, these values are, respectively, 0.953 and 0.900. As a consequence, with the same number of turns and the same flux, a three-phase motor would have, at its terminals, a voltage 1.06 times as great as that of the two-phase motor. It could obviously carry the same current, and consequently the input and the output would be approximately 6 per cent greater for a three-phase motor than for the same core wound for three-phase. Of course in practice the two motors would be rated the same, and the two-phase motor would in consequence operate at a slightly higher temperature. To make up for the lower voltage per winding, the current would be increased 6 per cent. The stator copper loss would therefore be increased in the proportion of 1.06^2 or approximately 12 per cent. Hence with the same output the losses of the two-phase motor are greater than those of the three-phase machine, and its efficiency is somewhat lower.

In a similar manner it could easily be shown that a six-phase motor would be slightly better than a three-phase one, and so on. It is easy to connect transformers so as to change a three-phase primary to a six-phase secondary. In fact all that is necessary is to connect the primaries to the line in delta, and bring out all six secondary leads to the motor. The neutral points of the three secondary windings may be connected together or not as desired. However, the complication is greatly increased by such an arrangement, since the switches, fuses, etc., all would have to be six-pole instead of three, and as a consequence the arrangement is not used in practice.

Determination of Copper Losses

It is customary to express the size of wires in circular mils, a circular mil being the area of a circle 0.001 in. in diameter. The circular milage of a wire is therefore equal to the square of the diameter in mils. The most convenient way of estimating the size of wire to carry a given current, is to allow a certain number of circular mils per ampere. The number to allow is of course based on experience with similar windings.

This also leads to a very convenient way of estimating the copper loss in a given winding. This is based on the fact that when we have such a current that the circular mils per ampere are 700, the loss in the copper is eight watts per pound. This figure of course varies with the temperature, and the above is correct only for 50 degrees Centigrade. The loss per pound will obviously be proportional to the square of the current, or inversely proportional to the square of the circular mils

per ampere. If then A_a denotes the circular mils per ampere, and G the weight of the wire in pounds, we have the copper loss

$$P_c = 8G\left(\frac{700}{A_a}\right)^2.$$

This formula is particularly useful in determining the losses in squirrel-cage rotors, since these have no definite resistance, or at least there are no definite points between which we can say the resistance is so many ohms.

Iron Losses

The losses in the iron of an induction motor consist of two parts: the hysteresis loss and the eddy-current loss. The former is proportional to the weight of the iron, and the frequency, and is dependent upon the maximum value of the flux. The loss varies with this maximum value in a more or less irregular manner, depending upon the composition of the iron. It is generally assumed that it is proportional to the 1.6 power of the maximum flux density. This assumption is merely a convenient approximation, but accords with the average conditions as well as any other known value. The loss due to hysteresis may then be expressed by the following formula:

$$P_h = yGfB_m^{1.6},$$

in which y is a constant, depending upon the quality of the iron.

Similarly the eddy-current loss is proportional to the weight of the iron, to the square of the frequency, to the square of the maximum flux density, and inversely proportional to the square of the thickness of the laminations. The eddy current loss can then be expressed by the following formula:

$$W_e = kGf^2B^2 \div t^2$$

in which as before k is a constant depending upon the conductivity of the iron.

In practice, iron for use in electrical machinery is tested in various ways. One of the simplest of these is to make the sheets to be tested the core of a transformer having a single winding. The winding is permanent, and is constructed in such a manner as to leave a narrow but long opening. The uncut sheets are then inserted in this opening and bent around to form a complete magnetic circuit. If the wire is of reasonable size, the loss in the apparatus when an alternating

current of the usual frequency is applied to it is practically all iron loss. This is measured by a wattmeter, and the voltage at the terminal is read at the same time. From this latter and the dimensions of the sheets, the flux density is readily calculated. The weight may be easily obtained, and from these data the loss per pound or per cubic inch can be at once computed.

It will be noted that the loss obtained is the total iron loss including both the hysteresis and the eddy-current loss. The same thing is true with most of the other methods. This is, in general, rather an advantage, since if the curves are prepared for the frequencies most used by the designer, he can at once obtain the loss per pound from the curves without computation. If the individual losses are desired, they can be obtained by making two tests at different frequencies.

It must not be thought, however, that the losses in the iron as above obtained can be applied without change to the computation of the losses in the completed motor. The losses in the actual machine will be found to be from three to five times as large as would be expected from the application of the values so obtained. Several factors contribute to this discrepancy.

The most important of these is perhaps the fact that the flux is not uniformly distributed throughout the metal of the laminations. It tends to take the shortest path, and this results in the flux density being greater in some parts of the iron than in others. It might appear at first sight that since the flux is less in other paths, the average loss would be the same. That this is not so appears at once when we consider that the hysteresis loss is proportional to the 1.6 power of the flux density, and the eddy-current loss to the square. This results in the uneven distribution, giving a much larger loss than would result from a uniform distribution.

Another factor tending to increase the losses is the filing of the slots. It is generally considered necessary to do this in order that the sharp edges may not cut the insulation, and that the coils may be easily inserted in the slots. The filing causes a conducting film to be formed on the inside of the slots, and this results in increased eddy-current losses.

In computing the iron losses, those in the teeth and those in the body of the iron should for the greatest accuracy be computed separately, since in general the densities are not the same. In practice, however, this is frequently not done, and an average value is taken for the whole motor. One method of doing this is to compute the whole weight of the stator as though it were solid, i.e., making no allowance for the

slots, and multiply by the factor corresponding to the flux density in the body of the iron. This procedure is allowable since the computation of this loss is in any case subject to large errors on account of unavoidable variations in the iron, and since the extra weight taken compensates roughly for the extra loss in the teeth. Fig. 66 gives average values for the iron loss per pound computed as above indicated. The curves were obtained from actual tests of completed motors. The iron was of good commercial quality.

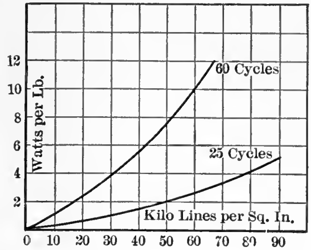


FIG. 66.—Hysteresis and Eddy Current Loss in Sheet Iron.

Rotor Iron Loss

In general it is assumed that the iron loss in the rotor of an induction motor operating near synchronism is zero. This is allowable since on account of the low frequency in the rotor the eddy-current loss almost disappears and the hysteresis loss does not exceed a few per cent of its value at the applied frequency. Of course this does not apply in the case of motors arranged to run at speeds far from synchronism, as in the case of the primary motor of two used in concatenation.

In certain cases however this assumption is far from true. This can perhaps best be shown by an actual example. Thus in Fig. 67 are represented the stator and rotor slots of a certain motor, as originally built. With these proportions the iron loss was about 6 k.w. This loss was about three times the expected value and the motor had a rise in temperature under full load of about 80 degrees Centigrade. This led to an examination of the design, and it was at once seen that the relation of the stator and rotor slots was such that a tooth in the rotor, such as A, would be at times so situated that almost all the flux from a stator tooth would pass through it, and at other times, when situated in the position of tooth B, there will be very little flux passing through it. It will be apparent at once that this construction will give rise to a high-frequency pulsation of the flux in the rotor teeth and a similar fluctuation in the stator teeth. The complete period of this pulsation

will be the time occupied by a rotor tooth in passing from one stator tooth to the next. The machine in question had eight poles, 54 stator slots and 115 rotor slots. Since it was operated on 25-cycle current,

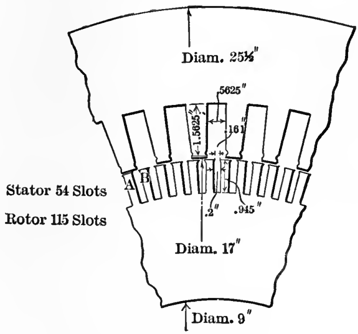


FIG. 67.—Stator and Rotor Slots.

the frequency of the flux in the stator was 25, that in the body of the rotor at 4 per cent slip, was one cycle per second. The frequency of

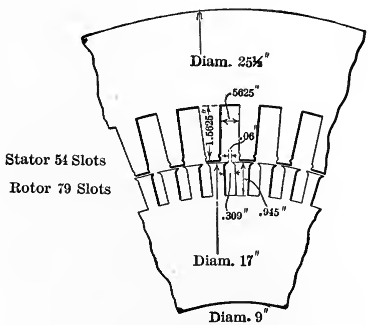


FIG. 68.—Stator and Rotor Slots.

the pulsation in the rotor teeth was, however, $54 \times 500 \div 60 = 450$ in the rotor and $115 \times 500 \div 60 = 960$ in the stator. It will be readily seen that even a comparatively small fluctuation of the flux at such a high frequency will give rise to considerable losses. This is particularly true of

the eddy-current component, both on account of the high frequency and on account of the filing of the slots.

It is of course true that a large proportion of the apparent fluctuation is prevented by the production of currents in the rotor bars. As soon as the flux through a tooth starts to change, a current is set up in the rotor bars surrounding the tooth in question, and this current tends powerfully to prevent the change of flux. This is true to any great extent, only in the case of a squirrel-cage rotor. It is perhaps impracticable to calculate the magnitude of this shielding, and in any event the lessened iron loss due to the shielding is in large measure compensated for by the increased copper loss in the rotor.

The remedy adopted in this case was to change the number of rotor slots to 79, adopting at the same time a more nearly closed slot as shown in Fig. 68. It will be apparent at once that the fluctuation will be greatly reduced. The test bore out this conclusion, as the iron loss after the change was only 2.2 k.w. instead of 6 k.w.*

Estimation of Heating

In designing induction motors for general use, the usual requirement as regards heating is that the rise in temperature under full normal load for an indefinite time shall not exceed 40 degrees Centigrade. To this is usually added the further requirement that under a load of 125 per cent of normal for two hours immediately following the full-load run, the rise shall not exceed 55 degrees Centigrade. Both the above are for a room temperature of 25 degrees, and are to be corrected one-half of one per cent for every degree that the temperature differs from 25 degrees.

In estimating the rise of a given machine, or in designing a machine so as not to exceed a certain rise, there are several different methods open to the designer. In the first place, such values are chosen for the circular mils per ampere, the flux density in the iron, etc., that the probability is that the temperature rise will be conservative. Thus in the average 60-cycle machine, if the circular mils per ampere are not less than 600, and if the flux density in the stator does not exceed about 40,000 lines per square inch, depending upon the quality of the iron, and if the ventilation is reasonably good, the probability is that the machine will not overheat.

* For a full discussion of iron losses, see "Calculation of Iron Losses in Dynamo Electric Machinery," by I. E. Hansen, Transactions of A. I. E. E., Vol. XXVIII, Part II, p. 993.

The heating is of course greatly affected by the peripheral speed of the rotor, the number and size of the ventilating ducts, the size and proportions of the machine, etc. The above is therefore valuable merely as a guide, and may be accepted as conclusive only in case the machine is similar to another in design, and this one has proven satisfactory in this respect. Thus if a 20-h.p. motor had a core length of 10 ins., it is certain that a 15-h.p. motor, having the same flux and current density, and having a core length of $7\frac{1}{2}$ ins., would operate at a slightly lower temperature. The fact that the temperature would be lower is due to the greater radiating surface in the second motor, in proportion to its losses. This fact is useful in that it allows us to dispense with the labor of computing the heating of all of a line of motors. If all are designed with the same constants, it is only necessary to calculate the heating of one motor of each diameter. The computation is of course to be performed for the motor of the greatest core length.

The opinions of different designers as to the best method of estimating the heating differ very widely. All of these methods are, however, based on the assumption that the rise in temperature is proportional to the number of watts radiated per square inch. The number to be allowed varies greatly with the peripheral velocity of the rotor, the dimensions of the machine, etc. The uncertainty and lack of agreement as to method arise mainly on account of the difficulty of estimating the actual radiating surface and the number of watts to be allowed per square inch. Thus some designers count as the radiating surface the so-called barrel surface of the machine, i.e., the interior area of the stator surface including the projecting part of the coils. On this assumption, from two to ten watts per square inch can be allowed, depending upon the peripheral speed of the rotor.

This method has not given good results in the experience of the author. In his opinion, a preferable method is to compute the total radiating surface of the machine. It is of course a matter of judgment whether or not all of a given surface, as the gap space or the interior of the air ducts, should be counted. It is his practice in general to count all of the exterior of the stator, the interior of the rotor, the ends of both the stator and the rotor, and one side only of the air gap surface and of the air ducts. The accompanying curve, Fig. 69, is the result of a large number of tests upon completed motors. The radiating surface was computed in the manner indicated. The losses considered were all the losses in the motor except that in bearing friction and the windage, and are the losses that will give a rise of 40 degrees Centigrade.

Curve A is for motors having plain rotors, and curve B for those having small wings. The values plotted were taken from a wide range of motors, and will be found to give reasonably consistent results in practice.

Where, however, a large number or motors have been built in the same frame, the author prefers another method. It is his belief, based upon a large number of tests, that in the case of induction motors, having frames of the conventional pattern, the heat-dissipating power of the machine depends almost entirely upon the dimensions of the frame, and upon the peripheral speed of the rotor. This seems reasonable, since the tendency of the manufacture is of course to use the smallest

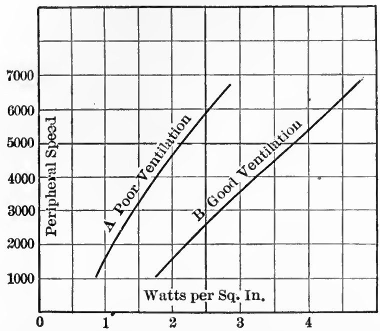


FIG. 69.—Allowable Watts per Square Inch of Core Surface.

possible frame, and in consequence the radiating surface of the active iron of the motor will not vary much in different machines in the same frame. In any case it cannot be doubted that the radiating surface of the frame itself is of the greatest assistance in keeping the motor cool. This method has been found to give much more consistent results than any other that has been tried, and has the further merit of being very simple to apply. Each new motor in a given frame is carefully tested for losses and the rise in temperature under full load for an indefinite time taken. The actual watts lost in the motor are then multiplied by 40 divided by the actual rise, and the result taken as the allowable total watts for the frame, at the given peripheral speed, without exceeding a rise of 40 degrees Centigrade. The number of watts that can be radiated without exceeding the allowable rise are

plotted with the peripheral speeds, and as soon as a sufficient number of observations are available, the curve representing the average of these points is drawn in. Used with discretion, this will give a reliable guide for all future designs in the same frame.

As a guide in case only one observation is available on a given frame, the author has found that the allowable watts for the frame can be represented by the following formula:

$$P=P_0(1+0.00016S),$$

where P represents the allowable power in watts for the frame at the peripheral speed S, expressed in feet per minute, and P_0 represents the allowable power in watts at standstill. The above is for machines only reasonably well ventilated and without wings on the rotor. In case of very good ventilation and a rotor provided with good-sized wings, the formula will be:

$$P=P_0(1+0.00032S).$$

An attempt has also been made to apply the formula to frames on which no tests are available. In this case the watts that can be dissipated are given approximately by the formula:

$$P=(1.3+0.00022S)A \quad \text{(without wings)},$$

and

$$P=(1.3+0.00044S)A \quad \text{(with wings)}.$$

The surface A to be taken is that of the total surface of a cylinder which would just cover the main part of the frame, the ends of the cylinder cutting through about the middle of the bearings. Such a formula can in the nature of the case be nothing more than a rough approximation, and wherever it is possible, the results of a test of the frame to be used, or of a similar one of nearly the same size should be used. The writer would place more reliance upon the computation of the watt per square inch of core and would use the above principally as a check.

CHAPTER X

FRACTIONAL-PITCH WINDINGS

THE proper pitch to use in the coils of an induction motor is of great importance. In most other classes of electrical machinery full-pitch windings are the rule. In the case of induction motors, however, the short-pitch winding is the rule and the full-pitch one the exception.

In brief, the advantage of the fractional-pitch winding is that the amount of wire in the end connections is reduced, and consequently the inductance and resistance of the stator or rotor, and the coils are distributed in more slots per phase per pole, giving essentially the same effect as though the number of slots were increased. The disadvantage

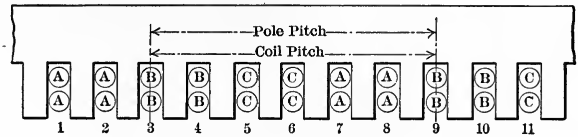


FIG. 70.—Full Pitch Coils.

is that a coil does not enclose all the flux from a pole and consequently the number of turns must be increased or a greater flux density employed. We will now consider these points more in detail.

In Fig. 70 is shown a section of a stator wound with a full-pitch winding. Fig. 71 shows a similar stator wound with a fractional-pitch winding. In both cases the winding is for three phase, and the number of slots is six per pole, or two per phase per pole. In Fig. 70 the pitch of the coils is 1 to 7, 2 to 8, etc. In Fig. 71 it is 1 to 6, 2 to 7, etc. The winding is such that there are as many coils as slots. Placing the coils in the slots with the pitches given, it will be readily seen that in the case of the full-pitch winding a slot will contain only coils of a certain phase. Thus, in Fig. 70, slots 1 and 2 contain coils of the *A* phase only. Slots 3 and 4 those of the *B* phase, and 4 and 5 those of the *C* phase.

In the case of the short-pitch winding, however, it will be seen that this is not the case. Thus slot 1 contains only *A* coils, but slot 2 has one *A* coil and one *B* coil. If we consider the coils belonging to any one phase, say *A*, it will be found that the coils of this phase are distributed one in slot 6, two in slot 7, and one in slot 8. It is evident that this will affect the motor to practically the same extent as though the motor were actually provided with three slots per phase per pole instead of with two, and the calculations are usually made as though this were the case.

If, instead of making the pitch in Fig. 71 1 to 6, it had been made 1 to 5, it will be readily seen that the windings of each phase would have been distributed in four slots per phase per pole. If an attempt were made to extend this further, it is apparent that a slot would be left between the two parts of the winding of a given phase, and no

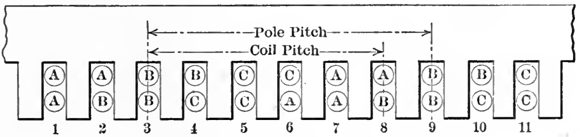


FIG. 71.—Fractional Pitch Coils.

advantage would be gained as far as the distribution of the winding is concerned.

The number of slots per phase per pole actually occupied by the windings of a given phase is sometimes called the equivalent slots per phase per pole. To determine this number, we divide the total number of slots by the number of poles and by the number of phases, and add to the quotient the number of slots by which the coils fall short of being full pitch.

That the amount of idle copper is reduced by using short-pitch windings is self-evident. The idle copper per coil, other things being equal, will be proportional to the coil pitch. As will be shown presently, the number of turns must be increased if the flux density is not increased. This of course tends to offset the advantage gained by the shorter length of end connections. However, the increase in turns for a moderate shortening in the coil pitch is very slight, while the saving in end copper is considerable. The determination of the pitch which gives the minimum amount of copper will be given later.

The calculation of the reduction in voltage per coil when short-pitch winding is used is very simple. In Fig. 72 let E and E represent the e.m.fs. generated in the two sides of a coil. The flux wave is assumed to be of the sine shape, so that the e.m.f. waves are also sine waves, and can consequently be represented by vectors. The angle θ represents the electrical angle between the two sides of the coil. For a full-pitch winding, the angle θ would of course be 180°.

FIG. 72.—E.M.F. Relations in Fractional Pitch Coils.

In the winding of Fig. 71, for example, since only five instead of six slots are spanned by the winding, the angle would be $\frac{5}{6}$ of 180° or 150°. In Fig. 72 line E_1 is the resultant of the two e.m.fs. E and E. It will be readily seen that its value is equal to $2E \sin \theta/2$. Since if the winding had been full pitch the resultant would have been $2E$, the back e.m.f. has been reduced by the use of the short-pitch winding in the ratio of $\sin \theta/2$ to one, or in an induction motor, if the flux density is to be kept the same, the turns per coil must be increased in the same proportion. The factor for the complete winding is of course the same as that for a single coil.

It is of interest to find the pitch which will in any case require the minimum amount of copper. In Fig. 73 let l represent the length of the straight portions of the coils, let P represent the pole pitch, and p the coil pitch. Also, let N_P represent the number of conductors which would be required if the winding were of full pitch and N_p the number actually required with the shorter pitch. If then k represent the ratio p/P, we have:

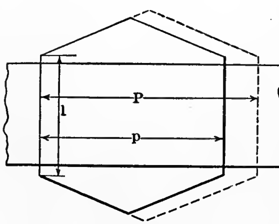


FIG. 73.—Relative Length of Copper for Full Pitch and Short Pitch Coils.

$$\frac{\theta}{\pi}=\frac{p}{P}=k \quad \text{or} \quad \theta=k\cdot\pi.$$

The length of one conductor will be $L=l+1.5p=l+1.5kP$, if we make the assumption that the length of the end connection is $1\frac{1}{2}$ times the coil pitch. This is a fair average of the ratio attained in practice. As we have just shown, the number of conductors must be increased as the pitch is made shorter, in such a proportion that

$$N_P=N_p \sin\frac{\theta}{2}=N_p \sin\left(k\cdot\frac{\pi}{2}\right).$$

The total length of conductor required is then

$$C=\frac{Np}{\sin\left(k\cdot\frac{\pi}{2}\right)}\cdot(l+1.5kP).$$

The only variables in this equation are C, the total length of copper, and k, the ratio of the actual pitch to the full pitch. To determine what value of k gives the minimum amount of copper, we must differentiate C with respect to k and place the result equal to zero. Thus

$$\frac{dC}{dk}=0=\frac{Np}{\sin^2\left(\frac{\pi}{2}k\right)}\times\left\{-l\frac{\pi}{2}\cos\left(\frac{\pi}{2}k\right)+1.5\sin\left(\frac{\pi}{2}k\right)-1.5Pk\frac{\pi}{2}\cos\left(\frac{\pi}{2}k\right)\right\}.$$

This readily reduces to

$$\tan\left(\frac{\pi}{2}k\right)=1.045\frac{l}{P}+\frac{\pi}{2}k.$$

Expanding $\tan\left(\frac{\pi}{2}k\right)$ we have:

$$\frac{\pi}{2}k+\frac{1}{3}\left(\frac{\pi k}{2}\right)^3+\frac{2}{15}\left(\frac{\pi k}{2}\right)^5+\frac{17}{315}\left(\frac{\pi k}{2}\right)^7+\ldots=1.045\frac{l}{P}+\frac{\pi}{2}k.$$

Reducing the coefficients of k it will be found that they are all (with the exception of the first) very nearly equal to 1.29. Assuming that they are constant we get

$$k^3+k^5+k^7+\ldots=0.813\frac{l}{P}.$$

This series may be replaced by $\frac{k^3}{1-k^2}$, giving as the final form

$$\frac{1.23k^3}{1-k^2}=\frac{l}{P}.$$

The ratio l/P is fixed from the construction of the motor, and substituting its value, we can compute the value of k. Since, how-

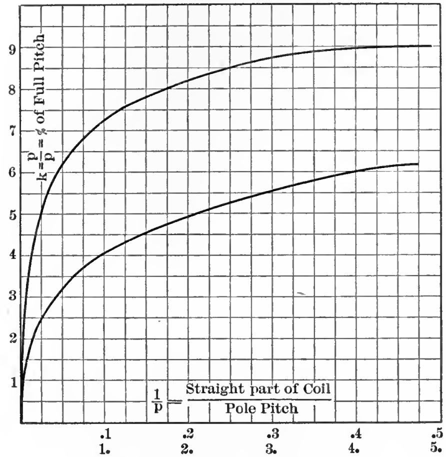


FIG. 74.—Curve of Values of "k."

ever, the equation is of the third degree in k, this is somewhat difficult. In Fig. 74, however, are plotted values of l/P and p/P or k. The most economical pitch can at once be found from the curve. Thus if the pole pitch were 10 ins., the core length 5 ins., and the coils project 1 in. on each side before starting to bend, the ratio l/P is 0.7 and the pitch should be 68 per cent of full pitch. If the core had been twice as long, or 10 ins., the most economical pitch would have been 74 per cent.

In applying the above, one should not lose sight of the fact that although the use of the pitch recommended will lead to the employment of the minimum amount of copper, the amount of copper, and

the number of wires in each slot is increased. This may lead to difficulty on account of the fact that the slot may not be large enough to hold the wire, or if a deeper slot is used, the depth of the iron back of the slots may have to be increased, and thus the greater amount of iron and the larger frame may offset the saving in copper. In such cases it is obviously wise to use a coil of slightly greater pitch, say of one tooth more than indicated.

The above demonstration applies equally well to the coils of an alternator or of a direct-current machine, provided the flux follows a sine distribution. As a matter of fact, the curves of flux distribution of such machines are usually flat topped, with frequently a considerable interval of zero flux. In this case, the gain by using short pitch coils is even greater than indicated above. In fact the pitch may be shortened to almost the width of the poles, without any appreciable loss in generated e.m.fs. In direct-current machines, however, short-pitch windings are not much used, since they virtually lessen the neutral zone.

Effect of Short Pitch upon Starting Torque

As has been previously shown, when the rotor is operating near synchronism, the rotor currents have a powerful effect upon the stator magnetism tending to cause the wave of flux to be of the sine shape and to rotate at uniform velocity, provided the applied e.m.f. is sinusoidal. At starting, however, since the rotor is at rest, this action is very much weaker. This is the case since the frequency of the current in the rotor, being the full line frequency, the setting up of the corrective currents is very much hindered. The result is that in both the squirrel-cage type, and in the wound-rotor type, the wave of revolving flux differs materially from the sine shape. The wave is not only distorted, but it also changes its shape from point to point. The *distortion* of the wave would in itself not lessen the starting torque, but the *fluctuation* of the flux does seriously reduce it.

The use of a short-pitch winding greatly reduces the distortion of the flux wave during the starting period, and hence considerably increases the starting torque. That this will be so can be readily seen, since in the short-pitch winding the phases overlap each other and consequently take off the corners of the flux wave. In fact, in the original United States patent on the short-pitch winding (issued to B. J. Lammé), it is this feature which is emphasized as being the principal advantage of the winding.

Stator and Rotor Windings

The subject of windings is a very extensive one, and to treat it in a comprehensive manner would require more time than can possibly be given to it here. The writer will therefore merely attempt to point out some of the distinctive features of the various windings, and would refer the student to any one of several excellent works that have appeared on this subject for further information.

Any direct current winding might be used on an induction motor. As a matter of fact, such windings are rarely if ever used, except in the case of a rotor provided with a commutator. The reason for this is

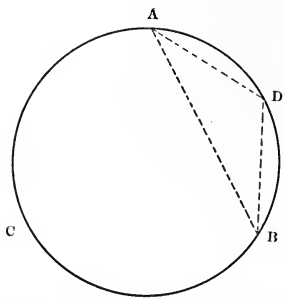


Fig. 75.—Comparison of Three-phase and Six-phase Windings.

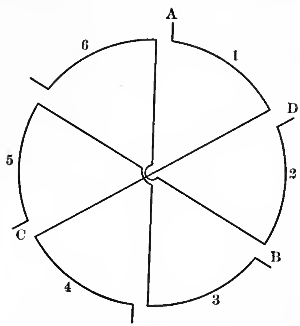


Fig. 76.—Six-phase Winding.

that such a winding would, for a given flux density and a given number of turns, require a lower e.m.f. at the terminals than would a winding of a different type, and in consequence the rating of the machine would be less. The reason for this will be readily seen from Figs. 75 and 76. If the number of conductors is large, the various e.m.fs. may be regarded as being short segments of a circle. Using the direct-current winding represented in Fig. 75 taps would be taken off from three points 120 electrical degrees apart as shown at A, B, and C for a three-phase winding. The counter e.m.f. developed by a given flux would be proportional to the chord of the circle AB. If instead of connecting in this way, we break the connection at D as shown in Fig. 76, and connect across the circle, the voltage developed by each phase for the same flux as before will be proportional to twice the

chord AD. This is obviously greater than AB in the ratio of 4 sin 30° to 2 sin 60° or 1.152. The winding shown in Fig. 76 will therefore give an output of approximately 15 per cent more than would a direct-current winding, used for a three-phase motor.

A little consideration will show that the winding of Fig. 75 is a true three-phase winding, while that of Fig. 76 is in reality a six-phase one. On account of the interconnection of the coils, only a three-phase e.m.f. appears at the terminals. The same principle applies to the case of a two-phase winding. For the best results the winding should be a four-phase one, and the connections so made that the terminal e.m.f. is two-phase.

Star or Delta Connection

It is obvious that the windings of Fig. 76 might be joined in either a star or a delta connection. Either will work with entire satisfaction and the choice between the two is largely a matter of convenience. Other things being equal, however, the star connection should be chosen. This is on account of the fact that with the star winding, since only 57.7 per cent of the total e.m.f. is applied to each winding, the number of turns will be less than in the delta connection. The size of the wire must of course be greater in the same proportion, and consequently there is no saving in copper, but the fact that the labor of winding, at least with small sizes of wire is less, and the fact that the amount of room required for insulation is less, cause the star winding to be preferred. This is particularly the case in high-voltage motors.

In the case of large low-voltage machines, the designer is often seriously restricted in his choice. Thus it might readily be the case that with a star winding, in order to obtain the desired flux density, 1.7 turns per coil would be required. The use of either one or two turns would be out of the question, the one giving too high and the other too low a flux density. If, however, a delta winding be used, the turns per coil should be increased in the ratio $\sqrt{3}$. Using this winding then the preferable number of turns would be 2.94 and three turns would be entirely satisfactory.

Pole Connections

It is of considerable advantage to a manufacturer to be able to change the voltage for which a machine is wound, without substituting new coils and reconnecting the entire machine. The

ability to do this readily may also be of the greatest value after the machine is in the hands of the agent or of the customer.

The voltages which are standard for induction motors are 110, 220, 440, and 550. To secure the above property, the machine would be designed primarily for 440 volts, and all of the stator windings would be in series. To reconnect for 220 volts, it would be necessary merely to divide the windings into groups corresponding to the various poles. All of these groups would be alike, with any of the common windings. The groups would then be connected together so that half of the groups corresponding to any phase would be in series, and the two halves would then be connected in parallel. In case the number of poles is divisible by four, the process can be extended to the case of a 110-volt winding, by connecting one-fourth of the coils in series and the four series in parallel. For 550 volts, a special winding is usually required.

In the case of large low-voltage machines, the number of turns per coil often comes out a very small number, and it may readily happen that it is impossible to find a number of turns with either star or delta connection that is entirely satisfactory. In this case, it is frequently desirable to arrange the winding so that the full voltage of the machine is impressed on only a part of the pole windings. The number of turns can then frequently be adjusted to a suitable value. Thus if 2.5 turns were required with a star winding, the corresponding number for a delta winding would be 4.33. Neither of these can be attained. By connecting only half the poles in series, the number of turns per coil to give the same flux density is increased to five in a star winding, and the difficulty is removed.

In a case like that mentioned above, where the number of turns per coil should be some integer plus one-half, as $6\frac{1}{2}$, $9\frac{1}{2}$, etc., the difficulty can in some cases be overcome by winding half the coils with a number of turns corresponding to the integer of the number desired, and the other half with one more turn each. Thus in the case referred to, half of the coils might be wound with two turns, and the other half with three turns each. This expedient is advisable only in case the number of slots spanned by the coils is odd. If this is the case, it will be found that when the top of a slot is occupied by a coil of the greater number of turns, the bottom coil of the same slot will have the smaller number. Hence the number of conductors in each slot will be the same, and if the coils are inserted in such a way that the upper coils in the slots have alternately the larger and the

smaller number of turns, the windings of the poles will be found to be symmetrical, and the winding can be connected up in any of the ordinary ways.

The Full Overlapping or Basket Winding

What is known as the full overlapping or basket winding, may be regarded as a special case of the above, in which the number of turns in one of the sets of coils has been reduced to zero. The number of coils is then equal to half the number of slots, and the same condition as given above holds as to the possible pitch.

The advantage of this winding is that it is somewhat easier to wind the coils, and the amount of insulating material is less, since the insulation between coils in the same slot is done away with. On the other

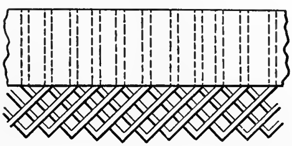


Fig. 77.—Diamond Shaped Coils.

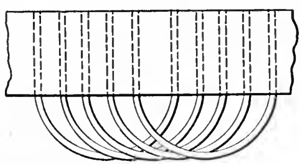


Fig. 78.—" Basket " or Full Overlapping Winding.

hand, somewhat more room is required for the end connections, the inductance of the end connections is increased, since the wires are bunched in larger groups, and the improvement in the dispersion coefficient by using short-pitch windings cannot be realized with this type of coil. The saving in the copper of the end connections by the use of the fractional-pitch winding, is of course retained. This type of winding is rarely used except in the case of small machines. Fig. 77 shows a winding employing the common type of diamond-shaped coils, while Fig. 78 shows a full overlapping coil.

The type of winding shown in Fig. 79 is known as a concentric winding or spiral-coil winding. Its great disadvantage is that it requires two or more forms for each winding instead of one. Moreover, it will be noted that some of the coils (in a three-phase winding), bend down on the ends, and those which bend down on one end are straight on the other. It will be seen at once that the coils cannot be completely formed before being placed in the slots, but that the coils must in general be cut in the middle of one end, and the cut ends of the

wires united one by one after the coil is in place. This results in the labor cost of such a winding being very high. Their principal advantage is that the ends of the coils may readily be so arranged that the different coils do not touch at any point. The coils may then be

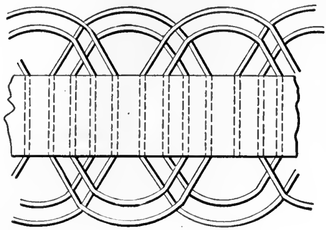


FIG. 79.—Concentric Winding.

easily insulated for high voltages, and the only point liable to break down is within the slots. This type of winding is therefore used principally in the case of high-voltage machines. Some of these objections do not apply to the case of a single-phase winding, and for this purpose concentric c6ils are commonly used.

CHAPTER XI

DESIGN OF A 50-H.P., 750-REV. PER MIN., 25-CYCLE INDUCTION MOTOR

In the present chapter an attempt will be made to show the application of some of the principles treated in the preceding chapters to the case of an actual motor. For this purpose, a machine of moderate size and operating at a favorable speed has been chosen. We should therefore expect to be able to design a motor of good characteristics and one that could be built at a moderate cost.

In designing such a motor, it should be kept in mind that the same frame with, if possible, the same slotting, should be capable of being wound for either two or three phases, and for the ordinary commercial voltages of 220, 440, and 550 volts. A machine of this size would rarely be wound for 110 volts. If required for 2200 volts, usually an entirely different design using larger slots, and probably a different core would be required. It is perhaps worthy of note that two-phase motors are by no means so frequently required as are three-phase ones, and this is particularly true in the case of 25-cycle motors. For this reason we will work out the design for three phases.

It is also generally advisable, as was previously pointed out, to build at least two ratings of motor of the same speed, frequency, etc., on the same size frame, and this should be kept in mind in selecting the appropriate diameter.

The first point to be determined is the output coefficient, as upon the value of this depends whether the design shall be a close or a liberal one. A great many considerations enter into the determination of this constant, such as the general policy of the company manufacturing the machine, *i.e.*, whether it is desirable to build a motor having very liberal characteristics and giving the best possible service to the customer, or to take the other extreme, one that can be built at the lowest possible cost and still meet the heating and other characteristics. If the machine is a special one, and only one of the rating is to be built, it is of course necessary to make the design much more liberal than

would otherwise be the case, since if the design should be unsuccessful, the percentage of loss would be much greater than would be the case if a great number were to be manufactured from the same design. A number of these points have been fully treated in the preceding pages.

In this case, the motor was one of a complete line of 750-rev. per min., 25-cycle motors, and from the results of the motors already tested and other experience it appeared that an output coefficient of about 38 was the best. The value actually used employing the nearest $\frac{1}{16}$ in. was 38.3. From the formula

$$38.3=\frac{50\times 10^6}{d^2 l 750},$$

we find at once that the value of d^2l is 1740. Since the machine has four poles, this is equivalent to saying that the product $t^2l=1080$, where t is the pole pitch and l the active length of iron, not counting the air ducts.

Our next problem is to determine the ratio of these two quantities. From the analysis of page 131, it will be seen that for the minimum amount of copper, if the winding is to be full pitch, we should have $t=\frac{1}{3}l$. This would give the dimensions $l=9.9$ and $t=3.3$, or the length of active iron would be 9.9 ins., and the diameter of the rotor would be 4.2 ins. It is apparent at once that this would lead to a motor having very poor facilities for ventilation, and one in which the power-factor would be very low. In fact it would be doubtless impossible to build a motor of these relative dimensions and using such a value of the output coefficient, and which would fall within the limit of 40 degrees C. rise. Without further consideration we can dismiss the idea of designing the motor on the basis of minimum copper cost.

We have found that to obtain the maximum value of the power-factor we should design the motor so that $t=(1.81=0.95S_o)l$. In this case it was decided to use slots one-third open and for this condition we find that $t=1.59l$. This leads to the values $l=7.55$ and $d=15.2$. These are reasonable values, and might be adopted. In this case on account of various considerations, such as the size of frames and punches available, and the fact that it was proposed to build both the 50- and the 75-h.p. motors with approximately the same flux densities and

on the same diameter of frame, it was decided to use a somewhat large diameter, and the value 17 ins. was determined upon. This gives at once 6 ins. for the value of the net core length.

As was previously shown, the air gap should be as short as is consistent with mechanical considerations. In the case of motors having a fairly large pole pitch, it is, however, not necessary to make the gap so short, in order to obtain reasonable values of the power-factor, as would be the case with motors of short pole pitch. In the present motor the length chosen was 0.035 in. A shorter gap could have been used if necessary.

The number of ventilating ducts is usually so taken that no part of the active iron of the motor is more than about one inch from a radiating surface. This leads to the use of two ducts in the present case. Each duct was made $\frac{3}{8}$ in. wide. In a larger motor, where the air would be forced to travel a greater distance past the laminations before being discharged from the motor, it would probably be better to make the ducts of a greater width. This is a matter to be settled by experience.

The number of slots chosen for the stator was 48, or 4 slots per phase per pole. This number also allows of winding the machine with the same slotting, as a two-phase machine, having six slots per phase per pole. Within reasonable limits, the greater the number of slots the better. Of course this could be carried too far, and the number of slots increased to so great a number that the percentage of insulating material would be so large that the advantage of the increased number of slots would be more than counterbalanced. It must also be kept in mind that an increase in the number of slots means, in general, a greater manufacturing cost.

Since an increase in the number of slots in the rotor will not result in so great an increase in the manufacturing cost as would the same increase in the stator, it is customary to use a greater number of rotor than of stator slots. It is also customary to use a prime number of slots in the rotor. This tends to prevent magnetic locking of the stator and rotor with consequently little or no starting torque. In this case, on account of various reasons, 110 was chosen as the number. Previous experience had shown that although not a prime number, it would give good results. The dimensions of the stator and the rotor slots are shown in Fig. 3.

The factors to be considered in determining upon the proper pitch of the coils have already been considered, and it has been shown (see

page 149) that the most favorable pitch is given by the formula, $l/P=\frac{1.23k^3}{1-k^2}$. This formula is represented in the curve of Fig. 74. The length of the straight part of the coil is in this case about $6\frac{3}{4}$ ins. $+1\frac{1}{4}$ ins. $=8$ ins., and the pole pitch is 13.4 ins. It is therefore seen from the curve that the best value of the pitch factor, as far as economy of copper is concerned, is 65 per cent. If as short a pitch as this had been chosen it would have caused the use of too many conductors per slot for the size of slot available, and on this account a somewhat longer pitch was adopted. The pitch actually chosen was from one to ten, or 75 per cent.

The next consideration is the number of conductors per phase to be used in the motor. This is a very important point and one upon which there is room for considerable difference of opinion. Other things being equal, upon the number of conductors depends the maximum output of the motor, the starting torque, and the point at which the maximum power-factor will occur. The principal points to be considered have already been treated. American practice in squirrel-cage machines has, to a certain extent, been standardized, and in the case of machines in which the power-factor will of necessity be low it is customary to alow a maximum output of at least 200 per cent of full load. In machines of large pole pitch in which the power-factor will be high, it is in the author's opinion advisable to allow a greater overload capacity than this.

This is the case in the present design, and a maximum output of about 250 per cent is desirable. Before the number of turns to give this can be computed, it is necessary to estimate the dispersion coefficient or the leakage factor in any of the ways already indicated.

This is the machine already referred to, and its leakage coefficient and dispersion coefficient have already been computed in various ways. Taking as the value of "C" the number 8.95 (as determined on page 119), using this number and substituting in the formula derived on page 120, we find as the maximum value of the k.w. input and consequently the approximate h.p. output,

$$P=\frac{25{,}800pE^2}{CbfN^2(\sin\gamma/2)^2}=\frac{25{,}800\times4\times440^2}{8.95\times25\times6\times352^2\times0.925^2}=140\text{ k.w.}$$

The winding used is to be connected in delta, hence the value 440 is taken as for voltage per phase. Eleven turns per coil are used, or the

number of conductors per phase is 352. As was previously explained, the winding would usually be star connected. In this case the delta winding seems to be preferable on account of giving a more suitable value to the maximum power input. This matter was fully discussed on page 152. The factor 0.925 or $\sin\left(\frac{135}{2}\right)^\circ = 0.925$, is used on account of the short-pitch coils (see page 147).

At full load and average efficiency and power-factor the motor will take approximately 62 amperes. The current per conductor in a delta-connected winding will be the line current divided by $\sqrt{3}$ or in this case 35.8 amperes. It will be found that 22 number 6 wires can be readily accommodated in the slot chosen. Using this size of wire we have $26{,}250 \div 35.8 = 735$ circular mils per ampere. In many designs this value is taken as low as 600. In this case, since the rotor loss is rather large the stator copper loss was kept low.

The maximum value of the flux density in the gap is found from the equation,

$$B_m = \frac{7.42 E 10^7}{ANf \sin \gamma/2} = \frac{7.42 \times 10^7 \times 440}{6 \times 13.4 \times 25 \times 352 \times 0.925} = 49{,}700.$$

The total flux per pole is given by

$$\Phi = 49{,}700 \times 13.4 \times 6 \times \frac{2}{\pi} = 2.54 \times 10^6.$$

The cross-section back of the slots, making 10 per cent allowance for the waste space between the laminations, is readily found to be 15.5 sq.in., and from this the maximum flux density in the iron is 81,600. This would be excessively high in the case of a 60-cycle motor, but is satisfactory in the case of a 25-cycle machine. This is so since for the same flux density the hysteresis loss will be reduced in the ratio of 25 to 60, and the eddy-current loss in the proportion of the square of this ratio. It is decidedly advantageous to have the density in the iron as high as possible, since by cutting down the cross-section of the iron, we reduce greatly the outside diameter of the motor, and consequently its weight and cost.

The flux densities in the teeth of the stator and the rotor are readily obtained by multiplying the maximum value of the flux density in the air gap, by the ratio of the slot pitch to the tooth section. The tooth pitch is, of course, to be taken at the air gap, and the tooth width at the point for which we wish to calculate the density. Some designers would increase the value thus obtained about 10 per cent to allow for

the spaces between the laminations. In this case, the density is found to be 122,000 lines per square inch at the root of the tooth, and 139,000 at the narrowest point. These values are about as high as is allowable both on account of the large losses at high densities, and to even a greater extent on account of the large magnetizing current required to force the flux through the teeth. This consideration is less important in the case of machines in which the power-factor is naturally high, i.e., in the case of machines of large pole pitch.

The iron loss is determined in the manner described on page 137. The weight of the stator iron is computed, no allowance being made for the teeth, and the average loss per pound for the given flux density determined from the curve. In this case the loss was taken as 3.8 watts per pound. The test of the machine after it had been built showed a total loss of 1325 watts, or 2.79 watts per pound. It might be again pointed out, that this is far greater than the loss which would be expected from a test of the iron with uniform flux distribution. For example, this particular iron showed under test a loss of less than one watt per pound, at the same flux density as that employed in the motor.

The determination of the current per rotor bar is perhaps most readily made by a consideration of the total sheet of current in the stator and that in the rotor. If the efficiency and power-factor were the same in both, these sheets would be the same, that is the arithmetical sum of all the currents in the stator and of all those in the rotor would be the same. While this statement would require some modification to be strictly true, it is nearly enough correct for our purpose.

To make use of this fact, we assume approximately the product of the efficiency and power-factor in the rotor, and calculate the current per stator conductor on the assumption that the power-factor and efficiency in the stator are the same as the rotor. The product of this current into the total number of stator conductors, gives the stator current sheet. This value divided by the number of rotor bars gives the rotor current per bar. In this case, we may take power-factor$\times$ efficiency$=0.92$. The current per stator conductor would then be 30.7 and current per rotor bar$=30.7\times352\times3\div110=295$. The size of the rotor bars was taken as 0.1574 in. by 0.787 in. This gives a cross-section of 513 circular mils per ampere.

The current per end ring is given by the formula

$$\frac{295\times110\times2}{8\times\pi}=2580.$$

The sheet of current under each pole divides and half passes in each direction. The number of bars under half a pole is in this case $\frac{110}{2\times 4}$. If the virtual current per bar is I the maximum current (assuming sinusoidal distribution) is $\sqrt{2}I$ and the average current will be $\frac{2\sqrt{2}}{\pi}I$. If the number of bars per half pole be n, the maximum current in the end ring will be $\frac{\sqrt{2}.2}{\pi}nI$ and the virtual value will be $\frac{2}{\pi}nI$.

The size of the rings was taken as $\frac{7}{8}$ in. by $1\frac{1}{4}$ ins. The material was cast copper. The cross-section is 539 circular mils per ampere. This is liberal, and the density might have been made somewhat higher without danger of heating in normal operation.

The active length of the rotor bars, i.e., the part carrying current is $9\frac{1}{4}$ ins., and the weight of the active rotor copper is readily found to be 41.2 lbs. The outside diameter of the ring was 16 ins. and the inside diameter 13.5 ins. The weight of both rings is then 31.6 lbs.

The losses in the bars at full load are given by

$$P_c=8G\left(\frac{700}{A_a}\right)^2=41.2\times 8\times\left(\frac{700}{533}\right)^2=568 \text{ watts.}$$

The loss in the end rings is likewise given by the formula:

$$31.6\times 8\times 3.65\times\left(\frac{700}{539}\right)^2=1550 \text{ watts.}$$

In this the factor 3.65 has been introduced, since the resistance of the cast copper used was approximately 3.65 times as great as is that of drawn copper as used in wires and rotor bars. This ratio of course varies somewhat in different samples, depending upon the amount of impurities, methods of casting, etc. The total loss in the rotor at full load is the sum of these two, or 2118 watts.

The magnetizing current is readily calculated from the formula given, but before doing so it is necessary to estimate what we may call the equivalent length of the air gap. This is made necessary on account of the fact that the flux, instead of being distributed in a uniform manner in the air gap is, on account of the teeth, distributed in a series of tufts. The density in these tufts is, of course, higher than would be the case if there were no projections, and consequently a higher m.m.f.

is necessary to force the flux across the gap. This is equivalent to having a longer air gap.

Methods of estimating the factor by which the length of the gap must be multiplied to correct for this tufting may be found in many of the text-books devoted to designing. In general we may say that there is no absolutely correct method of estimating this factor. As a matter of fact, in the case of the induction motor, it is not essential that we have such a method, since on account of the shortness of the gap, variations in the length of the gap are sure to occur and change so greatly the magnetizing current that an exact method would be superfluous. Perhaps as simple a method as any is to assume that all the lines of induction on leaving the stator teeth pass in a straight line directly across to the rotor. This assumption will be only approximately true even in the case of an induction motor where the air gap is very short, and would be far indeed from the truth in the case of an ordinary direct-current machine.

In the present case, the slot pitch in the stator is 1.113 in., and since the slot opening is 0.25 in., the exposed iron per slot is 0.863 in. The ratio of the slot pitch to the exposed iron per slot is therefore 1.29. In the same way the factor for the rotor is found to be 1.11, or the factor by which we must multiply is the product of these two or 1.43. Using this for the no-load current, we get

$$i_0=\frac{0.243Bmd}{N_1\sin\pi/2}=\frac{0.243\times 49{,}700\times 0.035\times 1.43\times\sqrt{3}\times 4}{352\times 0.925}=12.85.$$

The actual test of the machine after construction gave a no-load current of 13.9 amperes. The magnetizing current or the wattless component of this was 13.6 amperes. This agreement is as good or better than can be expected in the regular course of manufacturing, especially since no allowance was made for the ampere-turns required to drive the flux through the teeth and other iron of the machine.

Using the factor we have just determined for correcting the length of the air gap, we find the equivalent length to be 0.035 in. $\times 1.43=0.05$ in. Using this, the value of σ, the leakage coefficient, is readily found from the equation, $\sigma=C\dfrac{d}{t}=8.95\times\dfrac{0.05}{13.4}=0.0333$. From this the maximum value of the power-factor is given by

$$\cos\theta_m=\frac{1}{1+2\sigma}=\frac{1}{1+0.066}=93.7 \text{ per cent.}$$

Having the losses in the rotor at full load, we can readily calculate the starting torque of the motor. We have found that the maximum power the motor is capable of taking is 140 k.w., and this corresponds to a starting current of 367 amperes. The ratio of the starting current to the full-load current is $367 \div 62 = 5.92$. The rotor loss will be approximately proportional to the square of the primary current, and consequently the loss at start will be $2118 \times 5.92^2 = 75{,}000$ watts.

This is the starting torque in synchronous watts. The torque in synchronous horse-power is this number divided by 746 or 101 h.p. We most frequently, however, desire the starting torque in percentage of the normal full-load torque. Since the actual full-load torque would be different for each motor of the same rating depending upon the slip at full load, it is customary to calculate the starting torque in percentage of the torque the motor wouid have to develop to deliver full load if it were to operate at synchronism. In the present case, this is found by dividing the starting torque in synchronous horse-power by the horse-power of the motor or 202 per cent.

The slip is readily obtained by dividing the loss in the rotor at full load by the output of the rotor, or 5.67 per cent.

The stator copper loss is readily obtained from the weight of wire and the circular mils per ampere, and is found to be 1116 watts. The friction loss can only be estimated from other machines built on the same frame and operating at the same speed. In this case we may assume it as being 500 watts.

We now have all the losses, and adding these to the output we obtain the input. The output divided by the input gives the efficiency. In this case it is 87 per cent.

The comparison of the fixed and variable losses is of great interest, since the maximum efficiency will be obtained when these two are equal. The fixed losses consist of the iron loss plus the friction loss, and amounts to 11,800 plus 500 or 2300 watts. The variable losses are the copper losses in the stator and rotor, or 1116 plus 2118, or 3234. The best efficiency, as can be readily shown, will be obtained at a load of 42.5 h.p., and at this load it will be 87.4 per cent.

The best point at which to have the efficiency occur, is influenced by a variety of factors. If the load is frequently a light one, it is of course advisable to have the point of best efficiency occur somewhat before full load. If on the other hand the motor is so designed that the efficiency at full load and at 125 per cent of full load are equal, the best efficiency occurring at some intermediate point, the over-load

capacity of the machine will be good. The truth of this will appear if we consider that since the efficiency is the same as full load and at 125 per cent of full load the losses at the two points will be in the proportion of four to five. Hence if the rise in temperature under full load is not more than 40 degrees centigrade, the rise under 125 per cent load will not be more than 50 degrees centigrade. While it would probably not be advisable to operate constantly at this rise in temperature, yet its application for a reasonable length of time will do little or no harm. From the standpoint of the manufacturer, this property allows him to guarantee excellent overload characteristics.

The standard overload guarantee is a rise of 55 degrees centigrade for a run of two hours at 125 per cent of full load, the overload run to be made immediately following a full load run of sufficient duration so that the motor has attained its final temperature. Hence a motor so designed that the efficiency is the same at full load and 125 per cent of full load, will do far better than this guarantee. On the other hand, in the case of a motor in which the best efficiency is attained at about 75 per cent of full load, the losses will run up to a far greater value on over load, and the rise in temperature will approximate that of the standard guarantee.

The rise in temperature of the motor must next be estimated, and this can be done in any one of the ways described. For example, in the case of the frame used, it was known from previous experience that at the peripheral speed employed (3340 feet per minute), approximately 4250 watts could be radiated without causing a rise of more than 40 degrees centigrade. Since it was the intention to use small wings on the rotor, about 25 per cent more than this could be radiated. The computed loss, less the friction of the bearings and the air friction, is 5034 watts. Hence the machine should be safe as far as heating is concerned.

As a check on the above the radiating surface of the core was computed counting one side of the air gap and one side of each of the ventilating ducts, thus,

$$(25.5^2 - 10^2)\frac{\pi}{4} + \pi 6 \times (10 + 17 + 25.5) = 2730 \text{ sq.in.}$$

This gives 1.84 watts per square inch, and from previous experience it was known that approximately 1.5 watts per square inch could be radiated without wings or perhaps 2.25 with the small wings employed. The actual rise under load was 29.3 degrees in the iron and 38.3 degrees

in the copper. The actual losses less friction were 5165 watts. This would indicate that the frame would radiate 6250 watts with a rise of 40 degrees. Dividing this by the radiating surface 2730 sq.in. gives the value of 2.29 watts per square inch. These values of watts for the frame and watts per square inch may now be plotted by the designer on his private curves, for future reference and as a check on his previous values. In this way, a mass of valuable information is gradually built up.

CHAPTER XII

SPECIAL TYPES OF MOTORS

In the following some of the special types of motors which have been developed to meet certain needs will be discussed. In order to keep the treatment brief, the motors mentioned will be only such as are manufactured in the United States.

Most of the motors which are here designated as special types, have been developed in the effort to combine the advantages of the squirrel-cage and the wound-rotor types. Where the service does not present difficulties in regard to starting torque, and variable speed is not required, the former is in every respect the better. Its cost is less, it is of more simple and robust construction, its power-factor will average from one to two per cent better. If constructed with no regard to starting torque, the efficiency will also be better. If from 150 to 200 per cent starting torque must be provided, a high resistance rotor will be required, and this will probably reduce the efficiency to no more or slightly less than that of a wound-rotor machine.

If on the other hand the proposed application is one which requires frequent starting under heavy loads, or if it is necessary to limit the demand for power from the line or if the speed of the motor must be readily capable of being varied through a large range, the wound rotor type is the one to be used.

The General Electric type L induction motor is one of the many attempts to combine the advantages of the wound-rotor and the squirrel-cage types. The stator is of the standard type, differing in no way from one that would be used with a squirrel-cage motor. The rotor, however, is provided with a three-phase winding and a three-phase starting resistor is mounted inside the rotor itself. Three brushes are arranged in such a manner that they make contact directly with the resistors. The brushes are movable from the outside by means of a rod passing through the hollow shaft. By pressing this into the rotor, the successive sections of the resistor are short-circuited, and the motor is thus gradually brought up to speed.

This type of motor is entirely self contained, and no external appliances are required with the exception of a line switch and the line fuses. The absence of an external starter compensates at least in part for the increased cost of the rotor construction over that of a squirrel-cage rotor. In fact, at the present time, the General Electric Company offer the two types for approximately the same price. This price is some 25 or 30 per cent lower than the cost of a corresponding wound-rotor machine with its starting resistor, and hence, in applications where it can be used, it has a great advantage over either of the usual types.

As an offset to these advantages, we have the fact that the rotor is of much frailer construction, and is consequently more liable to mechanical injury or electrical breakdown. As was pointed out, the power-factor is lower than that of the corresponding squirrel-cage motor, and the pull out point, and hence the overload capacity, is less. We have also the disadvantage that the radiation of heat from the rotor is somewhat impeded by the resistors mounted there. The efficiency of the two types is about the same, the internal starter motor having perhaps some slight advantage.

This form of motor can not be used for work where variable speed is required. On account of the fact that the resistors are mounted in the rotor, it would be impossible to get rid of the heat generated if the motor were to be used under the condition of full torque with all or part of the resistors in circuit.

The Wagner Type BW Induction Motor

The Wagner type BW induction motor combines the advantages of the wound rotor and squirrel-cage types of motor. That is, it has approximately the starting and running characteristics of the G. E. type L motor but performs its functions automatically. Fig. 81 is a general view of the motor, and Fig. 82 shows somewhat the construction of the rotor.

The stator presents no particular features, being of the same construction as would be used in a squirrel-cage machine. The rotor, however, has a winding resembling that of a direct-current machine, the ends of the coils being carried out to the bars of a commutator. The winding is so connected that during the starting period a portion of the winding opposes its e.m.f. to that of the remainder. Suppose, for example, that the coils are so connected that one-third of the coils oppose the action of the remaining third. The result will be the same

as though the motor were wound with one-third of the turns actually employed, and a resistance equal to the resistance of the remaining two-thirds of the winding were connected in series with it.

As soon as the motor attains a certain predetermined speed, a centrifugal weight acts to press a short-circuiting device against the commutator. In this way, each of the coils is short-circuited and the motor acts in much the same manner as a squirrel-cage machine.

Although a commutator was mentioned as being used in connection with the short-circuiting of the machine, it should not be understood

Fig. 81.—Wagner Type of BW Induction Motor.

that the commutator is used in the ordinary way. It serves merely as a convenient means of connecting the coils in the two ways mentioned. No brushes are employed and the device is in no sense used as a commutator. The construction of the centrifugal device is the same as that employed on the well-known single-phase motors of this company's manufacture, except that the brushes and brush mechanism are omitted. The short-circuiting device is the same.

The only accessories required with this type of motor are a three- or four-pole main line switch and the corresponding fuses. The action of the motor is entirely automatic. To start, all that is necessary is to close the main line switch. At the proper speed, the centrifugal

device will operate and short-circuit the rotor. To stop, the main line switch is opened. The motor will therefore take care of itself in case the line voltage is interrupted from some cause, and later restored without warning. The motor is well suited for distant control as the main line switch may be located as any convenient point and the motor started and stopped from that point. It is likewise well adapted to the operation of automatic pumping systems where the circuit is opened and closed by the movements of a float or by similar means.

The starting characteristics of a 35 h.p., 1800 rev. per min., three-phase 60-cycle motor of this type as furnished by the manufacturer are shown in Fig. 83.

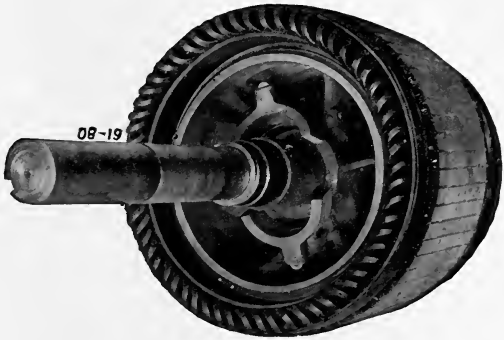


Fig. 82.—Rotor of Wagner Type BW Induction Motor.

Let us assume that one of these motors has such a rotor resistance that the slip at full load would be $3\frac{1}{2}$ per cent. At standstill if it were of the usual construction and full-load current were passed through it the rotor loss would likewise be approximately $3\frac{1}{2}$ per cent. This is on the assumption that the ratio of the stator and rotor currents is the same at standstill as at full load, which is nearly the case. Under these circumstances the starting torque would be the same as the rotor loss or $3\frac{1}{2}$ per cent. In the case of this motor, however, since one-third of the winding is opposed to the other two-thirds, we may consider that it is equivalent to a motor with one-third as many rotor turns, or with a given stator current the rotor current will be three times as large. Since the loss is proportional to the square of the rotor current, the loss will be nine times as large or the starting torque will be $9\times3\frac{1}{2}=31\frac{1}{2}$ per cent

with full-load current. If, however, the motor takes say three times full-load current when thrown on the line, as is the case with the motor, the characteristics of which are shown here, the starting torque will be increased in the proportion of the square of three or it will be 283 per cent. This is approximately the same as that of the motor shown.

On a curve of speeds and torque, not reproduced here, the actual slip of the motor is given as 9 per cent instead of 3½ per cent. This discrepancy is probably due to two causes. In the first place the short-circuiting device undoubtedly introduces some resistance into the rotor circuit and thus causes the slip to be somewhat greater than would

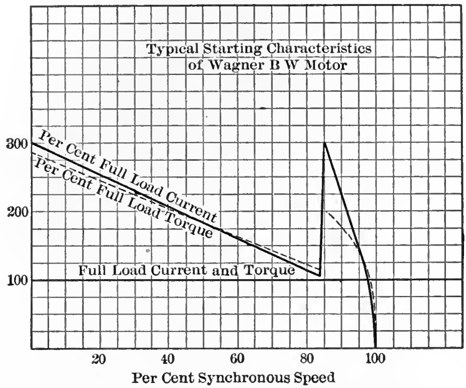


FIG. 83.—Starting Current of Wagner BW Motor.

otherwise be the case. The writer has no means of estimating the importance of this effect, but doubtless the increase of resistance is small.

In addition to this cause of increased slip we have the fact that during starting we have a more or less disturbed condition of the flux. This is due to the fact that the turns through which the current is in the wrong direction do not exactly offset the inductance of the third of the winding to which they are opposed. The effect is much the same as though the ratio of turns when short-circuited to the turns when not short-circuited were less than 1 to 3. Thus the rotor current and the starting torque are somewhat reduced.

As is apparent from the diagram, the current decreases as the motor approaches synchronism, and if the resistance remained the same a condition would soon be reached where the torque would be so much reduced that the motor would cease to accelerate further. The centrifugal device should be so set that before this point is reached the rotor winding is short-circuited. This short-circuiting should, however, be done as late as possible as the rush of current when the centrifugal device operates will then be less.

The principal merits of this device have already been mentioned. Its principal defects as compared with a squirrel-cage motor are that the efficiency and the power-factor are both appreciably lower. In addition the motor is by no means so simple, and is of course more liable to accident. That the power-factor is lower is apparent from the fact that the winding, although it is short-circuited, still presents many factors in common with the wound-rotor type of machine. The latter, as we have seen, has a leakage coefficient about 33 per cent greater than a corresponding squirrel-cage motor. This type of machine is intermediate in its characteristics, and its coefficient may perhaps be 15 per cent greater. Thus in the example given, the power-factor is 92.5 per cent at its maximum value. With an improvement of 15 per cent in the leakage coefficient, this value would be raised to about 93.3 per cent. The difference is not striking in this case since a four-pole motor of this rating is inherently of high power-factor.

That the efficiency is low follows from the fact that it is necessary to allow a considerable slip in order that the starting torque may be great enough. Thus the full-load efficiency is given as 83 per cent. The full-load loss is consequently 17 per cent. Of this the rotor copper loss accounts for 9 per cent, leaving only 8 per cent for the stator copper loss and the iron loss.

In comparison with the wound-rotor type of machine, this machine is somewhat inferior in that the starting torque is not adjustable. That is, the motor must take its maximum current at the start even though the load be very light. This is, of course, not a very serious objection. One of more importance is the fact that the torque drops off materially as the speed increases. If the starting torque required were say 150 per cent and this torque had to be maintained up to full speed, it would be necessary to set the centrifugal device so that the winding would be short-circuited at 65 per cent of full speed instead of at about 85 per cent. This would of course give rise to a large rush of current at the moment when the short-circuiting device operated. Moreover, a load

requiring nearly the maximum starting torque of the motor at all speeds could not be started at all by this type of motor. The conditions mentioned are not of frequent occurrence, but if present would require the use of a wound-rotor machine.

The Burke Induction Motor

The Burke Electric Company manufacture an interesting type of squirrel-cage induction motor in which the changes from the usual type are made apparently not for the sake of obtaining improved operating characteristics but for the sake of an improved construction from a

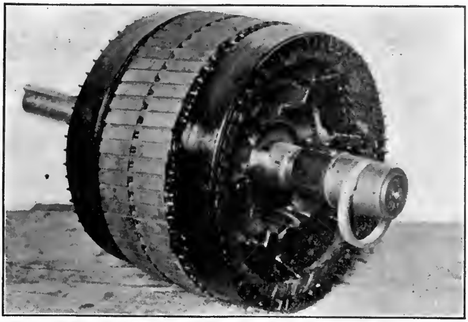


Fig. 84.—Rotor of Burke Induction Motor.

mechanical standpoint. Like most of the other motors here described, the changes are in the rotor. A general view of the rotor is shown in Fig. 84, and the construction of the rotor bars will be apparent from Fig. 85. Instead of using the usual bar winding, the rotor conductors are made by cutting a slit in a wide bar of copper, and pulling this out into the familiar form of a diamond shaped stator coil. These are then inserted in the slots in much the same way as is done in the case of a stator winding, and are secured by wedges.

The advantage of this type of winding is that since neither soldered or bolted joints are used, it is practically impossible to burn out the rotor winding. On the other hand, there is the disadvantage that on account of the greater length of the end connections, the leakage coefficient of the machine is greater than would be the case with a squirrel-

cage winding. As has been explained, this results in lower power-factor, slightly lessened efficiency and a considerably lower starting torque and pull out point. Using bars of pure copper in this way, it would also appear difficult in many cases to obtain sufficient resistance in the rotor to give the desired starting torque. Of course this could be obtained by reducing the section of the bars to the proper point, but this has the disadvantage that the heat developed on account of the

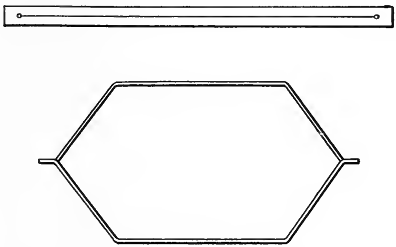


FIG. 85.—Rotor Coil of Burke Induction Motor.

rotor loss is largely concentrated inside the slots. It is of course preferable to have this heat generated in the end rings, as it is much more easily radiated from the rings than would be the care if it were compelled to travel some distance before coming to a surface exposed to the air.

SQUIRREL-CAGE CRANE MOTORS

Motors of this type were formerly used to a considerable extent for some classes of hoisting work, or in general,work where heavy starting torques and to a certain extent variable speeds were required. The rotor, while of the squirrel-cage type, was made to have a high resistance so as to give a large starting torque. This rotor resistance was so adjusted that at standstill, about three-fourths of the maximum possible torque would be developed. This would give a fairly large value of the torque whatever the speed.

The slip at full load is of course great. Moreover, with the large slip the motor will slow down nearly to standstill before reaching the pull out point. These facts lead to a convenient method of varying the speed. This is done by charging the applied voltage by means of an auto transformer. If with a given torque the voltage is reduced,

since the flux is reduced in the same proportion, the current must increase. This of course leads to an increased rotor loss and a consequently greater slip.

The principal objection to this method of operation is the low efficiency on account of the large rotor loss and the great heating in the rotor. On the other hand, it is very simple and would appear well adapted to conditions where these drawbacks would be of minor importance.

The Mill Type Motor

Since the introduction of the electric motor into steel mill practice, there has arisen a demand for a motor of somewhat different characteristics than those required for general service. Steel mill work is typical of a class of power work where everything must be subordinated to the mechanical reliability of the motor. The great object of the steel man is to get out the tonnage. The cost of the power used, while of course a considerable item, is still small in comparison with the value of the product. The efficiency of the motor is therefore of minor importance in comparison with its reliability. These considerations have led to the development of a special and well known form of direct-current motor, known as the mill type.

In general, these motors are entirely enclosed so as to prevent the possibility of damage from the fall of heavy articles on them. The frames are usually of cast steel. The bearings are of massive construction and the size of the spiders, shafts, etc., is such that the motor will not be injured by a possible jamming of the rolls or other machinery, causing the motor to be brought to rest almost instantly. In short, every endeavor is made to avoid the possibility of breakdown, either on account of accident or ignorant usage.

The demand for induction motors of the mill type has not been so great as for the direct-current motors. This is, of course, on account of the comparatively recent introduction of the use of polyphase alternating currents into service of this kind. The greater simplicity of the alternating system, both as regards the generators and the motors, is a powerful factor tending to increase the use of alternating current for just such service. On the other hand, the principal drawbacks are the fact that the induction motor is not so well adapted to variable speed work as the direct-current motor, and the fact that the starting torque, even with the use of a wound-rotor machine, is by no means so great as that of the direct-current series wound motor. In spite of these unques-

tioned drawbacks, the induction motor is gaining ground, and there is a demand for a motor in which the strength of the construction is emphasized, even with some loss in the matter of efficiency and power-factor and of course with an attendant increase in price.

In Fig. 86 is shown a motor of this character built by the General Electric Co. and known as their type M motor. At present, it is built for 25-cycle service only and in sizes from 30 to 150 h.p. As regards strength of parts, and general construction, it corresponds to what has been said regarding this type of machine. Special attention has been given to ease of repairs, and provision has been made so that the stator or the rotor can be readily removed in case of accident and a spare part substituted. The motors are built only in the wound-rotor type, and every part, including the slip rings, is completely enclosed.

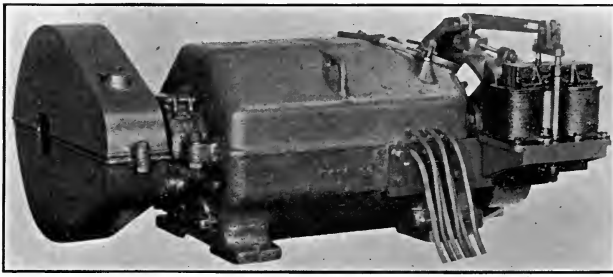


Fig. 86.—General Electric Induction Motor, Mill Type.

The rating of these motors is on much the same basis as that of traction motors. They are guaranteed to carry full load for one hour with a temperature rise of from 65 to 75 degrees centigrade. They will also carry 25 per cent overload for ten minutes without injurious heating, and 100 per cent overload momentarily. Their continuous rating is 25 per cent of their rated power.

It might be pointed out that is it particularly difficult to design an induction motor of the completely enclosed type for continuous operation, on account of the necessarily rather large iron loss and the fact that even at no load the stator current will be about 25 per cent of the full load current. It is therefore difficult to get the losses low enough so that the machine can radiate the heat developed, even though the rating be made very low for the frame used.

The Westinghouse Type MS Motor

The Westinghouse Company also manufacture a mill type motor known as the MS motor. Its general appearance is well shown in Fig. 87. The design follows in general the lines already indicated as regards strength, size of parts, etc. The motors are, however, usually operated as open motors, but can be partially or entirely enclosed if necessary. The rotors used are of the squirrel-cage variety. The peculiar construction of the end rings is illustrated in Fig. 98, and is described in connection with the descriptions of various end ring constructions.

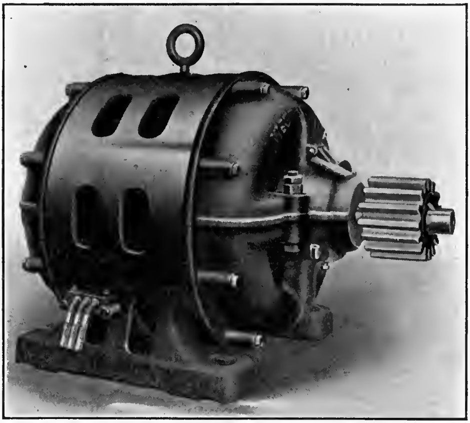


Fig. 87.—Westinghouse Mill Type Induction Motor.

Typical Constructions of Various Induction Motor Parts

A brief review of some of the various elements that enter into the construction of the induction motor may be of interest. The author has attempted to select examples which show in each case the latest practice of the manufacturer, whose product is described.

Frames. The most common type of frame is perhaps that illustrated in Fig. 88, which is a view of a standard Fairbanks Morse Induction Motor.

The body of the frame is usually constructed with numerous openings, so as to afford a free exit for the heated air from the interior of the motor. The bearing arms are frequently constructed with a view of affording protection to the stator winding where it extends beyond the core. Bearing arms are in universal use instead of the pedestal type of frame, formerly much used for motors of both the alternating-current and direct-current type. The difficulty experienced in machining such

Fig. 88.—Fairbanks Morse Induction Motor.

a frame together with the added weight, has caused its abandonment except in the very largest sizes of motors.

In Fig. 89 is shown a type of frame used by the Fort Wayne Electric Works in its medium sized motors. This is known as the skeleton type frame. As will be readily seen, it is similar to the frame just described except that the portion of the iron between the ribs is omitted. This results in saving a great deal of weight, and need not weaken the frame if the ribs are made sufficiently strong. It is claimed that by this construction, the laminations are directly exposed to the air, and are therefore enabled to radiate their heat more effectively. While this is doubtless true, it must be remembered, on the other hand, that the motor is deprived of the radiating effect of that part of the frame which has been cut away. The material of the frame being in direct contact

with the laminations becomes nearly as hot as the latter, and is consequently of great value in keeping the motor cool. It is a question for the manufacturer to decide, whether or not the amount saved by using the open type of frame will if invested in more laminated iron and copper, lead to a motor that is, on the whole, more satisfactory.

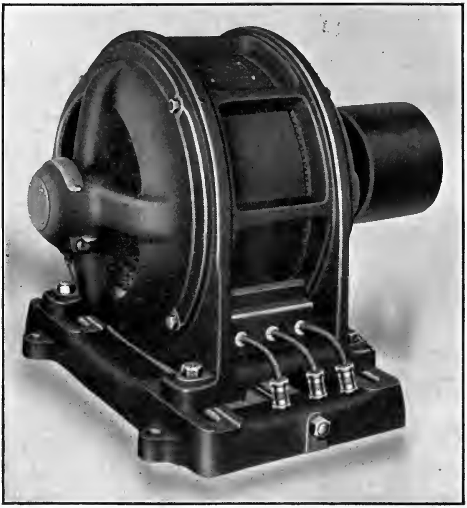


Fig. 89.—Fort Wayne Induction Motor, Skeleton Frame.

Fig. 90 shows a General Electric motor with what is known as the riveted type of frame. When using this type of construction, the laminations and the two end plates are assembled in the correct relation, and are then bound together permanently by rivets. This form of construction leads to a very light motor.

In all of these forms, the end brackets are held by either four or eight bolts. This allows the brackets to be rotated through either 90 or 180 degrees, and consequently allows the motor to be mounted on a wall or ceiling.

In the case of motors of large size, a box frame with pedestal bearings is commonly used. A frame of this character, used with a Westing-

house motor of 650 h.p. is shown in Fig. 91. In the case of still larger motors, the stator and the pedestals may be mounted directly on the

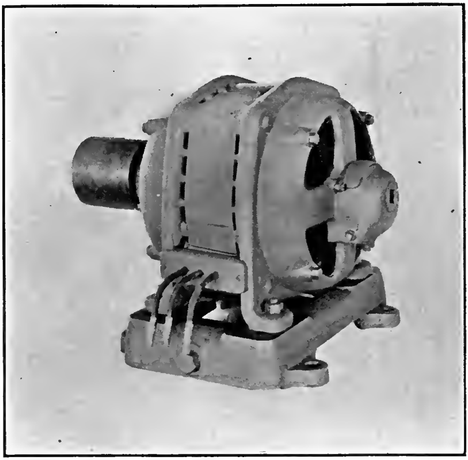


Fig. 90.—General Electric Motor, Riveted Type Frame.

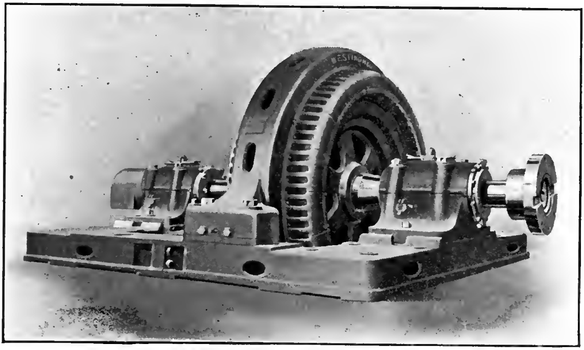


Fig. 91.—Heavy Duty Westinghouse Induction Motor.

foundations. A 6000 h.p. slow speed 6600-volt General Electric motor, in which this form of construction is used, is shown in Fig. 92.

The enclosed type of induction motor is in less demand than corresponding sizes of direct-current motors, since the induction motor is better able to withstand the effect of dirt, dampness, etc. Occasionally, in case of extremely bad conditions, particularly if the load is of an intermittent nature, they may be desirable. In Fig. 93 is shown a completely enclosed General Electric motor. The annular radiating rings are provided to assist in getting rid of the heat generated. Such motors are generally designed for operation with intermittent loads. If required for continuous operation, it would be necessary to design

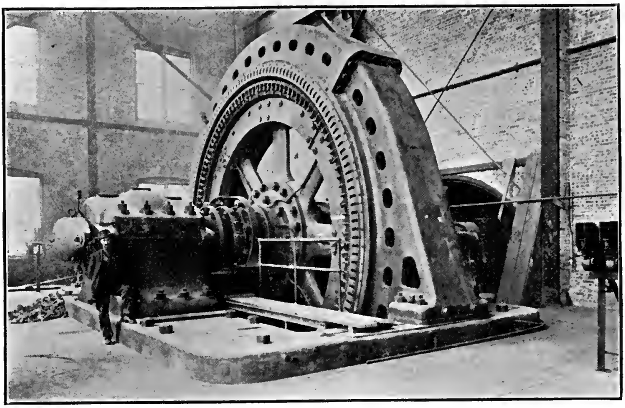


FIG. 92.—6000 H.P. 6600 Volt General Electric Induction Motor in Rail Mill, Gary, Ind.

them with very low flux densities, and consequently with small output for a given weight.

Rotor Construction. In Fig. 94 is shown the rotor of a Westinghouse motor. This illustrates a common form of construction, using solid end rings, fastened to the rotor bars by means of bolts. In addition, to preserve the joint from oxidation and improve the contact, the joint is also soldered. Instead of bolts to hold the bars to the ring, rivets are sometimes used.

Fig. 95 shows the form of rotor construction used by the Fairbanks Morse Co. The bars are relatively thin and a deep notch is formed in both ends of each bar. The bars and rings are first assembled and

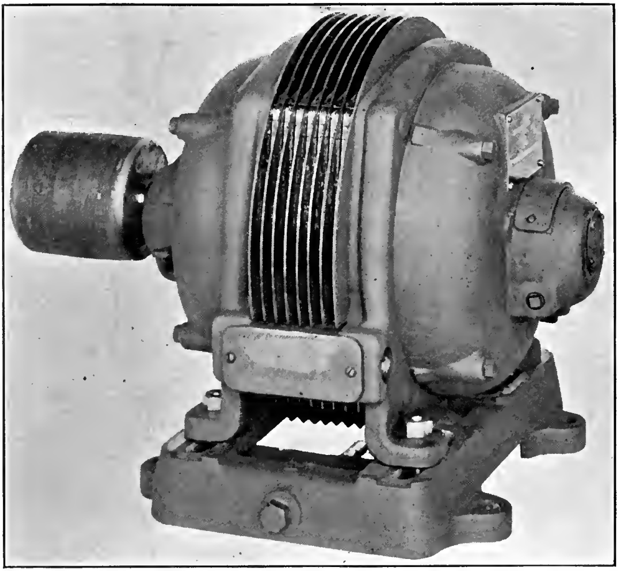


FIG. 93.—General Electric Motor Completely Enclosed.

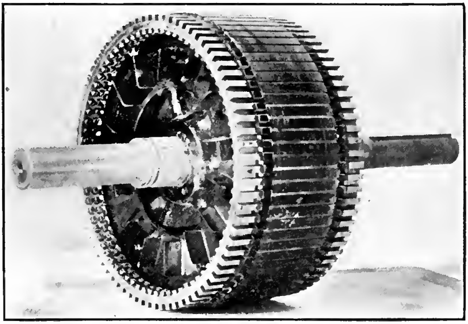


FIG. 94.—Squirrel-cage Rotor of Westinghouse Motor.

securely soldered by dipping each end of the rotor. The rotor is then placed in a lathe, the ends of the rotor bars are turned off, and a small notch is formed on the ends of the bars. A brass ring, having a projecting lip, bored to the same diameter as the notch just mentioned, is then forced on each end of the rotor bars and secured by a number of screws passing through the retaining ring and into the end ring. The whole is then dipped in solder for added strength and to seal the ends of the screws, and is then trued up in the lathe.

The type of construction used by the Burke Electric Co., in which there are no joints in the rotor conductors, has already been described on page 173.

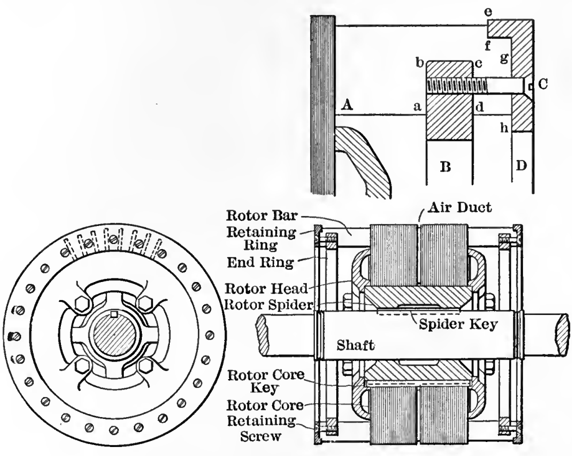


FIG. 95.—Construction of Fairbanks-Morse Squirrel-cage Rotor.

A type of soldered end ring largely used by the General Electric Co., is shown in Fig. 96. As will be apparent from the illustration, each ring is made up of several parts. This affords excellent facilities for ventilation, when the rotor is in operation, and the cross-section of metal can in consequence be small. While the motor is at rest or is revolving slowly, the rings are only slightly cooled by the fanning action of the air, and they consequently become much hotter than would solid rings of a correspondingly larger cross-section. The rings may be made of a material having a large temperature coefficient. They will

consequently increase considerably in resistance during the starting period, thus giving a good starting torque, but will quickly cool off as soon as the rotor reaches full speed. They will thus act automatically to give greater starting torque than would be possible with the same slip with the ordinary type of solid end ring construction. It is of course evident that care must be taken to keep this rise of temperature within suitable limits, as otherwise the solder might be melted from the rings.

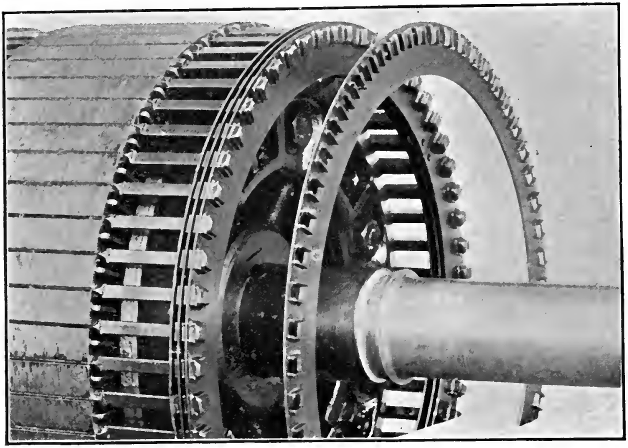


FIG. 96.—General Electric Squirrel-cage Rotor.

In Fig. 97 is shown the rotor of a Crocker-Wheeler motor of somewhat similar design. In this, however, the rings instead of being continuous around the whole circle, consist of short segments. In assembling these segments, they are placed on the ends of the rotor bars in such a manner that each one overlaps the one ahead of it. Looking down on the rotor, the rings present a spiral appearance. It is claimed for this method of construction that it obviates the tendency present in the form previously described for the inner ring to take more than its share of the current. In this way, the danger of the inner ring becoming much hotter than the outer ones is avoided.

In Fig. 98 is shown a form of end ring used by the Westinghouse Co. on its mill type motors. Their machines are designed to withstand the roughest sort of usage, and in consequence every effort is made to

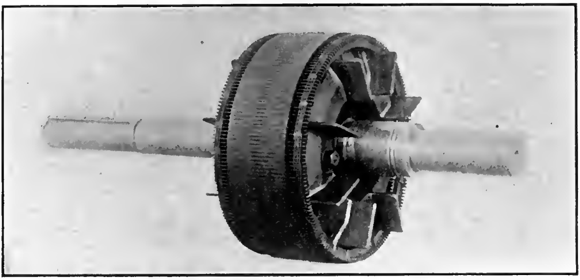
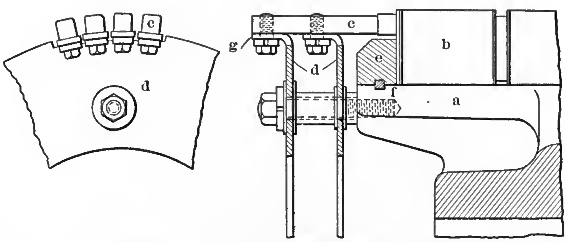


FIG. 97.—Squirrel-cage Rotor, Crocker-Wheeler Induction Motor.

make the construction as simple and durable as possible. As shown in the illustration, the end rings consist of punched sheets of resistance



a-Rotor spider b-Laminated core c-Conductor d-Resistance rings. e-End ring. f-Key. g-Lock washer.

FIG. 98.—Construction of Rotor of Westinghouse Mill Type Motor.

metal, and are provided with lips turned at right angles, and adapted to make contact with the rotor bars. They are secured to the latter as shown, and are further supported by insulated studs, screwed into the rotor spider. No solder is used.

General Notes on the Selection of Motors. Speed

From the standpoint of the motor itself, the higher the speed the better. Of course in the majority of cases, the speed is fixed, at least to a certain extent, by considerations external to the motor. If these are such that the speed of the motor must be fixed at some rather low value, the manufacturer is of course compelled to do the best he can under the circumstances. It must, however, be kept in mind that the motor for the lower speed will be higher in price, and at the same time lower in efficiency, power-factor, pull out point, and starting torque.

On account of the lower speed, a larger frame must of course be used, thus increasing the cost. Also, since the amount of active material is greater, if as is usually the case, this active material is worked at the same flux densities and current densities, these losses will be greater or the efficiency will be lower.

The fact that the power-factor of any induction motor is less than unity, is due to the fact that the motor requires a current lagging 90 degrees behind the e.m.f. to maintain the magnetic flux across the air gap and through the iron. The value of the magnetomotive force is proportional to the length of the air gap, and the number of poles. It is entirely independent of the size of each of the poles. The slow-speed motor requires a greater number of poles than one of higher speed, and consequently requires a greater magnetomotive force, and has a correspondingly lower power-factor. To overcome this to a certain extent, the manufacturer is forced to use the shortest possible air gap, resulting in a motor which will not withstand so much wear before the rotor will come into contact with the stator, as would have been the case if a more conservative design could have been adopted.

This objection has far more weight in the case of 60-cycle motors than in the case of those for 25 cycles, since in the latter the number of poles for a given speed is far less than in a 60-cycle machine.

Frequency

The question of the frequency to be employed is in the majority of cases settled beforehand by other considerations. In any event the question has been fully discussed in Chapter IX, and we may merely make the statement that in general, taking everything into consideration, a frequency of 60 cycles is the best when the majority of the motors are to be of small and medium size, and that 25 cycles is the better when the motors average about 100 h.p. or over.

Size of Motors

Every effort should be made to select the smallest possible motor for the work in hand. It is not intended to imply by this that motors so small that they will be regularly overloaded should be used, bu that for example a 50-h.p. motor should not be used, if a 35-h.p. machine is of ample size to carry the load. In direct-current work it is often the best practice to use large motors, in the expectation that future growth will so increase the load on the motors as to render them ultimately of the correct size. With direct currents, this merely involves a somewhat greater initial investment and a slightly lower efficiency as long as the load is light. To offset this it is argued that in addition to the motors being ultimately of the correct size, there will in the meantime be less liability of trouble since the motors will be of excess size, and it will not be necessary to interrupt production to change to a larger size.

Of course no one can deny that these arguments have weight also in the case of an alternating-current installation. The fact that induction motors have a relatively low power-factor when operated at partial loads, should act powerfully to prevent the use of a larger motor than is necessary. As a matter of fact, the best power-factor is usually found between 100 and 125 per cent of full load, and in some cases, may be found at an even larger overload.

The bad results attributed to the use of underloaded induction motors operating at low power-factors, are noted, not so much in the motor itself, as they are in the generator supplying the power. Low power-factor results both in loading the generator up to its full current capacity long before its power limit is reached, but what is perhaps more serious, acts powerfully to spoil the regulation. This is of course more apparent in cases where the generator power rating is relatively small compared with that of the induction motors fed by it. For example, if half the full load generator rating were required for incandescent lamps, and the other half by induction motors, the effects of the poor regulation caused by lightly loaded motors would not be so apparent as would be the case if all the power were required by induction motors.

There are several ways in which this poor regulation shows itself. If lamps are fed from the same generator, the variations of the voltage will result in very poor service. This will be especially the case if the lamps are on the same circuit as the motors. As regards the motors, the effects will be apparent in insufficient starting torque, and a tendency of the motors to pull out on overloads which they would readily carry

if the voltage were normal. Since both the starting torque and the pull out point are proportional to the square of the voltage, this may easily become serious. Moreover, the trouble is cumulative. That is, the low power-factor causes the voltage to drop. This would of itself call for a proportionately larger current to supply the same power through the motors, but in addition, on account of the lowered voltage, the motor is virtually changed to a machine of smaller rating, and on account of the large load, it may work at still lower power-factor. This in turn again reduces the voltage, and so on. Thus it will be seen that trouble of this sort once started may readily grow, and even become so bad as to render the operation of the plant at its rated load entirely impracticable.

Of course, this condition of affairs may be helped to some extent by the installation of an automatic regulator, but on account of the fact that it requires a short time for the regulator to act, it may not be as useful in overcoming the difficulty as might at first sight appear probable. The safest course is to install generators of ample size and hence of good regulation, and motors of the smallest size possible with the load to be carried. If in the future it is necessary to use motors of higher rating, this may frequently be done without great expense or inconvenience by interchanging the heavily loaded motors for others which are of larger size (using other motors already installed) and in turn substituting others for those used in this way.

Squirrel-cage and Wound-rotor Machines

In every installation the question of whether to employ wound-rotor machines or those with squirrel-cage rotors must be determined. In general we may say that the wound-rotor machine is a necessary evil. For operation at full speed and after the machine is in motion, the squirrel-cage machine is in every respect superior. The one possible exception is in the matter of efficiency. If the squirrel-cage motor is required to develop starting torque with a limited starting current, it is necessary to use a rotor of high resistance, and this may in certain cases reduce the efficiency to that of a wound-rotor machine or even to a lower value. In regard to price, power-factor, pull out point, ruggedness of construction, and freedom from liability of trouble, the squirrel-cage machine has an unquestioned advantage.

It is in the matter of starting torque and availability for variable speed work that the wound-rotor machine has an advantage. To

develop 100 per cent starting torque with a squirrel-cage machine will require approximately 300 to 400 per cent of full-load current, and this current is unfortunately lagging almost 90 degrees behind the applied e.m.f. As a consequence, the voltage regulation of the circuit is seriously interfered with, and it is usually necessary to take the starting current from back of the fuses, thus leaving the motor without protection during the starting period. On the other hand, the wound-rotor machine will develop 100 per cent torque with but little if any more than 100 per cent of full-load current. Other torques require current in proportion. Moreover, this current is of a much higher power-factor than that of the squirrel-cage motor during starting, and hence the line disturbance is even less in proportion than would be indicated by the ratio of the currents.

This large starting current of the squirrel-cage motor may or may not be of importance, depending upon various conditions external to the motor. Thus if the generator supplying the energy be of large size in comparison with the motors to be started, the disturbance at starting will be small. In order that this may generally be true, the generator should be capable of furnishing approximately ten times the full load current of the largest motor. With such a proportion the starting of an induction motor of the squirrel-cage type should cause a fall of voltage of not much more than 10 per cent.

If the motors are larger relatively to the generator than the figures just given, say the generator is capable of furnishing twice the full-load current of the motor, we may still secure satisfactory service providing the load at starting can be made very light, or providing the large fall in voltage during starting is not objectionable. For example, it might be desirable to start a large motor of this character only in the morning and at the noon hour. It would be entirely practicable to start this motor first and then start the smaller motors or other devices when convenient. The large fall in voltage would then not affect any other apparatus on the line.

The ratio given above, that is the k.v. amp. rating of the generator double the h.p. rating of the motor, is about the limiting ratio that can be employed at all, if it is necessary to develop full-load torque during starting. To do this requires that an amount of power equal to the full h.p. rating of the motor be wasted in the rotor. In order to get the current corresponding to this loss into the rotor requires a loss in the stator of nearly the same amount. In some cases this stator loss may be even greater than the rotor loss, although in standard motors

it will in general be less. On the average we shall not be far wrong if we assume that to start a squirrel-cage motor under full-load torque requires approximately twice full-load power to be furnished to the motor. Hence in the assumed case, the full k.w. output of the generator would be required. Moreover, this power is supplied at a low power-factor and hence the full-load *current* rating of the generator would be greatly exceeded.

If it is desired to install a generator and a motor to take the entire output of the generator, it will be necessary to employ a wound-rotor machine or to make arrangements so that the motor can be started with little or no load. On the other hand, motors with phase-wound rotors may be freely employed up to the rull rating of the generator.

These facts in regard to the better starting characteristics of the wound-rotor machine, indicate that it should be used in all cases where starting is of frequent enough occurrence to constitute an appreciable proportion of the whole running time. Such applications are electric elevators, electric hoists, crane motors, motors for electric traction, etc.

Some of the above involve work which requires variable speed. For this class of work the squirrel-cage motor is out of the question, and we are forced to employ the wound-rotor machine. Unfortunately, we have nothing on the market at the present time that at all corresponds to the adjustable-speed, direct-current shunt motor. With this type of machine, it is possible to set the speed at a certain value by changing the shunt field current, and after this adjustment the speed will be but little changed by subsequent changes in load. We have available at the present time only one alternating-current machine that possesses at all these characteristics. This is the induction repulsion type single-phase induction motor. Even this machine possesses these characteristics to only a slight degree compared with a direct-current motor, that is, the range of speed adjustment is small. There is little or no doubt that a polyphase motor having the same characteristics could be developed, and undoubtedly such a machine will be offered as soon as there is a sufficient demand for it. It would have a commutator and brushes, and means would be provided for impressing various voltages on the commutator. In addition to its variable speed features, such a motor could be made to have a power-factor of nearly unity, or even to take a leading current. (See page 86).

Unfortunately, the slip ring type of motor does not possess these features; that is, as the load is varied, the speed will vary within wide limits, particularly if the resistance of the rotor circuit is considerably

increased to bring the speed down to a fraction of synchronism. If this is the case and the load is removed, the speed will increase to nearly synchronism. It is therefore necessary to keep this fact in mind in using the wound-rotor type of induction motor for variable speed work. This fact forces the employment of direct current in order to be able to utilize adjustable speed motors. In many cases, however, it is possible by the use of mechanical speed changing devices to get rid of the necessity of the variable-speed motors.

Group or Individual Drive

The choice of the type of motors to be used in any installation is bound up with the question of the use of the group drive or of individual drive. The group drive is to a certain extent an outgrowth of the older system of driving through a line shaft. When such a system is to be changed to an electric drive, the most obvious and simple way of making the change is to split the shafting up into such sections as is most convenient, and drive each of these sections by means of a comparatively large motor. In addition to the fact that this method reqiures the minimum amount of change from the older system of drive, it has several advantages of its own. The total horse-power of the motors installed will be much less than would be required if each tool were to be provided with a motor of its own. This condition arises largely from the fact that we can in general count on not having all the machines connected with a given motor operating at the same time. Thus the total h.p. output required at any given time will be less than would be the case if each tool had its own motor, since in this case it would obviously be necessary that the total rating of the motors be equal to the total rating of the machine tools.

On the other hand, it may frequently happen that it is necessary to operate only one small machine out of the entire group. To do this it will of course be necessary to operate the motor connected to the group. Since the efficiency of a motor at light load is small, this results in a waste of power.

A typical case of a condition where a group drive would be indicated is that of a number of sewing machines. These are usually all of the same size, and the nature of the work is in most cases such that all of the machines are in operation at the same time. The cost of the one motor to drive the group is obviously less than would be the cost of individual motors for each machine. The efficiency is also

much higher. Other examples, such as groups of small speed lathes, automatic screw machines, etc., will readily suggest themselves.

On the other hand, a case calling for the use of individual drive would be that of a large boring mill, planer, or lathe, which was in rather intermittant use. To operate a number of machines of this character using group drive, would obviously require that a long section of shafting be kept in motion almost all of the time. This would of course result in a large loss of power, both in the shafting, and on account of the motor being frequently under-loaded.

In case the group drive is the one selected, the motor to be chosen will almost invariably be of the squirrel-cage type. This is evidently the case, since the service does not require a large starting torque, and the speed is constant. If the amount of shafting used is so long that a large starting torque is necessary, it would in most cases be preferable to use shorter sections so as to reduce this loss, or to go to the individual drive.

If individual drive is chosen, the best motor in the majority of cases will still be the one with squirrel-cage rotor. The tools can almost invariably be started up without load and consequently the starting torque will be small. If variable speed is required, the wound-rotor machine must be used, but it must also be kept in mind that, as was pointed out, it is not possible to adjust a motor for a given speed and expect it to retain that speed no matter what the load may be. Thus if a lathe were equipped with a motor of this character and it were operating at low speed with a deep cut, and the tool should run out of the cut, the motor would speed up with disastrous consequences when the tool again entered the cut. In the machine shop, as far as the driving of tools is concerned, there would therefore be little use for the wound-rotor machine.

For the operation of electric elevators, for crane motors, and for hoists, the phase-wound machine is the only suitable one. This arises on account of the frequent starting and the large torque required in these applications. In most cases, also, these motors would be rated for intermittent service.

Motors for Centrifugal Pumps

The application of induction motors to centrifugal pumps is becoming of very frequent occurrence. For this service the squirrel-cage motor is the one almost invariably recommended. This is certainly

the correct motor to use if the head under which the pump is to work is constant at all times. If, however, as is frequently the case, the head varies from time to time, the wound-rotor machine should be used. This is on account of the fact that the speed of a centrifugal pump should be rather accurately adjusted to the head under which the pump is operating. If this is not done, the pump is very inefficient. It might be argued that we might as well waste the power in the pump as in the regulating resistance of the motor, but it can readily be shown that less power input to the motor will be required if the motor and pump are slowed down by the increase of resistance in the rotor circuit than would be the case if the machine were allowed to operate at full speed. This is true even in spite of the loss in the regulating rheostat.

CHAPTER XIII

THE SINGLE-PHASE INDUCTION MOTOR

In the preceding pages we have shown that in a polyphase motor with an infinite number of phases, if each of these phases is supplied with an e.m.f. of the proper magnitude and phase, and if the applied e.m.f. is sinusoidal, a sinusoidal band of flux rotating with synchronous speed will be set up. The same result will also follow if we have only a finite number of phases, say two or three, provided the conductors of each phase are distributed in proportion to the ordinates of a curve of sines. If this is the case, the sheet of current set up by the current of each phase is harmonic. In a practical motor, it is impracticable to distribute the conductors in this manner, and we frequently content ourselves with arranging the conductors and consequently the current in bands of uniform strength. In the single-phase motor, however, some attempt is often made to approximate a sinusoidal distribution. Frequently a short-pitch winding is used in polyphase motors, and this has the effect of making the bands overlap and consequently the current approaches more nearly to an harmonic distribution. In either case the tendency is to set up a non-harmonic band of flux, as shown in Figs. 8 and 9. The currents produced in the rotor conductors, particularly in the case of a squirrel-cage rotor, act powerfully, however, to prevent any change of flux, and the actual rotating magnetic field is very nearly harmonic.

The single-phase induction motor may be considered as a special case of the polyphase motor. If a two-phase motor is operating without load and one of the phases is opened, the motor continues to run at almost exactly the same speed as before. The only apparent change is that the nature of the hum emitted by the motor changes slightly in character. If an ammeter is used to measure the current taken by the machine (in this case principally magnetizing current) it will be found that the current in the phase still connected has approximately doubled, as has likewise the power taken by this phase. The total current and the total power are nearly unchanged.

If when operating in this manner, a voltmeter be applied to the idle phase it will be found that nearly the full line voltage is present there, and a further investigation will show that this e.m.f. differs 90 degrees in phase from the line voltage. The application of test coils at various angles with the active phase would show the presence of nearly the same e.m.f. regardless of the position of the coil, and an angular difference of phase corresponding to the angle of the coil with the stator winding. This experiment proves the existence of a rotating magnetic field, and a fuller investigation would show that this field is harmonic in its space distribution and rotates with uniform angular velocity.

In the light of what has previously been said, it is almost self-evident that this will be the case. The flux tends to assume such a distribution and value that the minimum cutting of the rotor conductors and consequently the minimum expenditure of power will take place. Each change of flux sets up a rotor current in such a position and phase as to tend to prevent the change of flux. With a rotor operating at synchronism, a uniform flux rotating at uniform velocity would not cut the rotor bars at all. Hence the flux tends to assume this distribution and velocity.

However, to produce the rotating magnetic field, in a single-phase induction motor it is evident that there must exist a component of m.m.f. at right angles to the axis of the stator winding. Since the stator can not carry a current in the proper position to produce this it must exist in the rotor. To produce this rotor current requires a change in the stator flux. Hence the field can not be of absolutely constant value at all times. The change however is slight.

The nature of the current required in the rotor to produce with the stator current a rotating magnetic field in the motor, will be apparent from Fig. 99. The four diagrams are drawn for successive values of the current taken at intervals of 30 degrees. The flux is shown displaced 30 degrees to the left for each change of 30 degrees in the current.

The solid line marked stator current represents the distribution of the stator current over the stator surface. The conductors are assumed to be arranged on the stator core in such a manner that the number of conductors at any given point is proportional to the sine of the angle corresponding to the point. The arc between two poles is of course taken as 180 electrical space degrees. The current is the same in all of the conductors and hence the sinusoidal curve represents the distribution of the current over the stator core. The curve of distribution is

stationary in space, but variable in magnitude, changing from a positive to a negative maximum, in accordance with the change in the current.

In order that an harmonic, uniformly rotating magnetic flux be

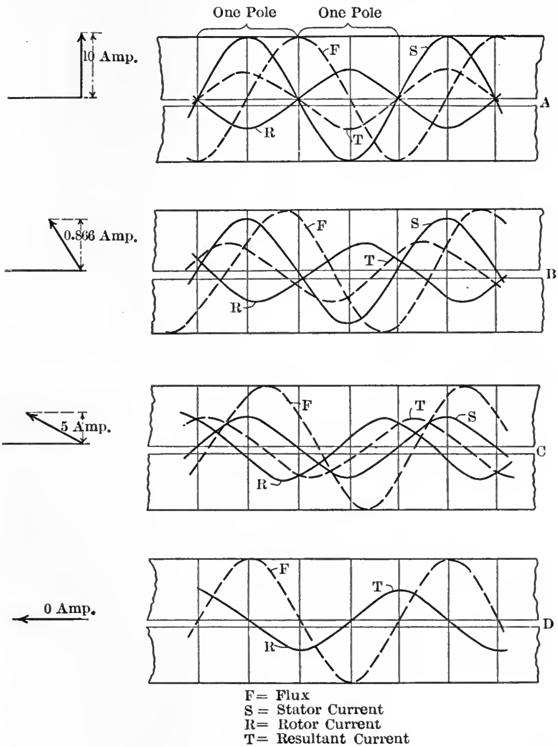


FIG. 99.—Current Distribution in Stator and Rotor of a Single-phase Induction Motor. Sinnusoidal Distribution of Stator Coils, Rotor Squirrel-cage.

maintained, it is necessary that a *resultant* harmonic band of current rotate uniformly around the stator. This resultant is due to currents in both the stator and the rotor. It is shown as a dotted line in each of the figures and is drawn 90 degrees ahead of the flux. The resultant

band of current is due to the algebraic sum of the current sheets in both the stator and rotor at any given point. The rotor current is then the difference between the resultant current and the stator current. It is shown by the curve marked rotor current. In order that the required rotor current may circulate, it is necessary that the rotor be of the squirrel-cage variety with many bars. With a phase-wound rotor, the current could not assume the exact values required at all points and the resulting rotating flux would not have exactly harmonic distribution.

A study of the construction of the diagrams will reveal the following facts:

A. The flux has harmonic distribution, is constant in magnitude and rotates uniformly in the direction of rotation of the rotor.

B. The stator current sheet is stationary in space distribution, and has harmonic variation in magnitude.

C. The rotor current sheet is harmonic in space distribution, of constant maximum value, and rotates backward, i.e., opposite to the direction of rotation of the rotor at synchronous speed. Its maximum value is half that of the stator current sheet.

It is apparent that at the time shown in Figure *D*, the rotor current sheet must be sufficient to force the total flux across the gap. Hence its value is the same as would be required in a second phase of the stator if one were present. Its space location is 90 degrees from the stator winding. The conclusion readily follows, that a single-phase motor requires twice the magnitizing current that would be taken by one phase of the same motor-wound two-phase with the same number of turns per phase. Similar reasoning would apply to a three-phase motor compared with a single-phase, or in general we may say that the volt-amperes required are the same whatever the number of phases. The same principle was explained in developing the formula for the magnetizing current of a polyphase motor. The foregoing may be considered as a proof that the same formula applies to the single-phase motor.

The curves of Fig. 99 are constructed on the supposition that the conductors on the stator core are distributed in such a manner that the number of conductors at any given point is proportional to the sine of the angle at that point, counting from some fixed point of reference on the stator core. It is hardly necessary to point out that such a distribution is not feasable in practice. In many motors, some attempt is made to approximate this condition by winding some coils with less turns than others. To do this requires coils of the concentric

type, as shown in Fig. 79. For this reason, and for the sake of symmetry, such coils are frequently employed in single-phase motors.

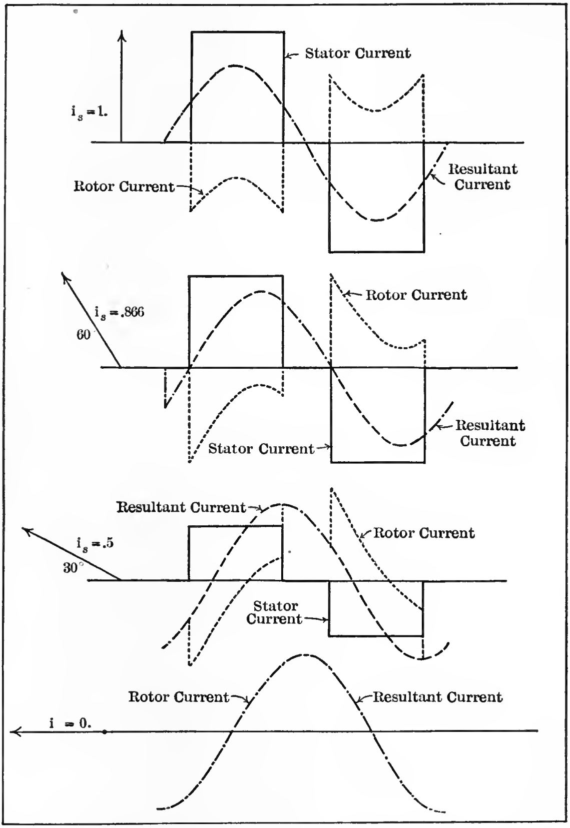


FIG. 100.—Current Distribution in Stator and Rotor of Single-phase Induction Motor. Stator Coils in 120 Degree Bands. Rotor Squirrel-cage.

The curves shown in Fig. 100 may be considered as an example of the extreme opposite condition. This represents the currents in

the stator and rotor of a three-phase motor, wound with full pitch coils, and operated on a single-phase circuit. The curve of distribution of the stator current is of course rectangular as shown. The resultant band of current is sinusoidal, and the rotor current is of the proper value to give, in combination with the stator current, the resultant harmonic band of current. As before, it will be seen that the rotor current sheet moves in the opposite direction from the resultant current sheet, but that it is now very much distorted from the sine shape.

These curves, like those of Fig. 99, are for the no-load condition. With the motor under load, the value of the rectangular stator current would be increased in proportion to the current. There would be added in the rotor a corresponding rectangular current distribution, almost exactly equal and opposite to the added stator current. In the case of Fig. 99 a corresponding sine distribution of current would be added. It is this added component, in connection with the component of the flux at right angles to the direction of the stator winding, that produces the rotor torque.

Returning to the ideal case of sinusoidal distribution, as shown in Fig. 99, instead of considering the rotor current sheet as a band of current rotating backward in space, we may perhaps gain a better idea of the phenomena involved if we separate the band of rotor current into two component current sheets, each stationary in space but varying harmonically in magnitude. The bands differ 90 degrees in time phase and are displaced 90 degrees in space. As we have already shown, the combination of two such stationary bands is equivalent to one rotating band. Each band of current may be represented by a vector as shown in Fig. 101. The resultant current sheet will be of constant value, and will rotate as shown.

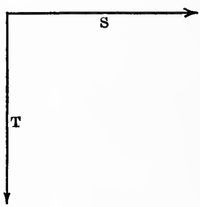


Fig. 101.—Vector Diagram of Current Bands in Rotor.

Under the condition of no-load and synchronous speed, we have just seen that we have in the stator a current sheet of double the value of the rotor current sheet. This sheet can be represented by a vector of double the value of one of the vectors representing the rotor current sheets and at an angle of 180 degrees with one of them. The relations are then as shown in Fig. 102, and the direction of rotation of the resultant of the three current sheets will be in the opposite direction to that of the rotor current sheet. The net result

then is that we may consider that we have three stationary current sheets, one in the stator and two in the rotor. One of the rotor sheets is directly opposed to and offsets half of the stator current sheet. This resultant then combines with the remaining rotor sheet to form a rotating current sheet of constant value. This rotating current sheet sets up a corresponding rotating flux sheet which is likewise constant in value and rotates in synchronism with the current sheet. The above applies of course to the no-load condition only.

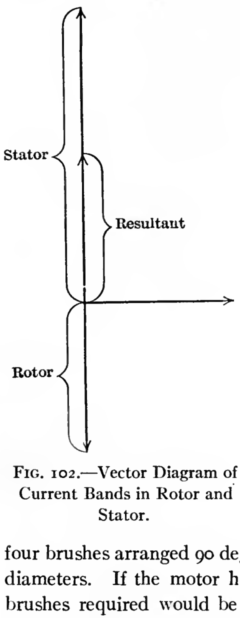


Fig. 102.—Vector Diagram of Current Bands in Rotor and Stator.

Fig. 103 represents a type of single-phase motor to which the preceding analysis particularly applies. The coil marked P represents the stator winding. The motor represented diagrammatically is of the two-pole variety, and of course in the actual machine half of the stator winding would be on each side of the stator core. The rotor is of the same construction as the armature of a direct-current machine. It has a commutator and as shown has in a two-pole machine four brushes arranged 90 degrees apart and short-circuited across both diameters. If the motor has more than two poles, the number of brushes required would be twice the number needed with the same armature and commutator used as a direct-current armature. For the sake of simplicity, we may regard the winding as being a ring winding. This is convenient since in this case the m.m.f. of either circuit is in line with the corresponding brushes, or in other words, if there is a current through either set of brushes the corresponding poles of the armature will be in line with the brushes used.

An armatue of this sort, if used in a polyphase field, will operate in much the same manner as would a squirrel-cage rotor. We have studied this action somewhat in Chapter VI. If used in a single-phase stator, it will likewise operate much as would a squirrel-cage rotor. The current forming the current sheet T will be through the brusher T and T', that forming the sheet S through S and S'.

There is another way in which we may consider the action of the

single-phase motor, and which is very convenient in developing the formulae of the motor. This method consists in brief in considering the *flux* as being resolved into components and considering separately the effect of each of these components. Thus in Fig. 103, we may assume that we have a flux in the direction TT' due to the resultant of the current through the stator coil and the rotor current in TT'. There is also a flux in the direction SS' due to the current through the brushes SS'. These two fluxes are of equal value but differ 90 degrees both in phase and in space. Their resultant is therefore a rotating magnetic field. The current in SS' is called the speed current and the corresponding field the speed field. In all the conductors each way from S to S' there will be generated an e.m.f. proportional to the speed and to the value of the flux in the direction TT' at the instant under consideration. This e.m.f. will then be in phase with the flux TT'. The wave will be sinusoidal if the flux variation along TT' is sinusoidal. This condition is usually assumed in theoretical discussions.

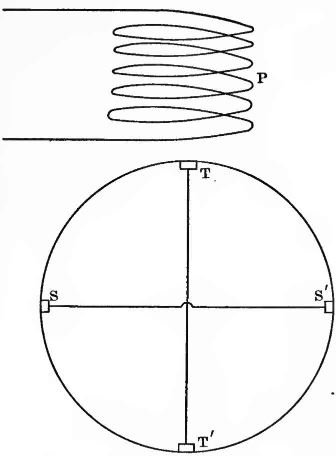


FIG. 103.—Single-phase Commutator Type Induction Motor.

The resultant current along SS' will lag nearly 90 degrees behind the generated e.m.f. since the rotor is highly inductive in this direction. The flux in the direction SS' will, except for the slight effective hysteresis, be in phase with the current. It is therefore 90 degrees behind the e.m.f. in SS' and likewise 90 degrees behind the flux in TT'.

The flux TT' is of nearly constant value, irrespective of the speed of rotation of the rotor. That this is so is apparent when we consider that the counter e.m.f. in the stator coil is nearly equal to the applied e.m.f. and hence the flux producing this e.m.f. must also be nearly constant to produce a constant back e.m.f. The difference between

the applied and counter e.m.f. is due to the resistance drop in the stator and the drop due to the local leakage reactance, i.e., that due to the lines which cut the primary coil out do not pass through the rotor. Notwithstanding these two drops, the counter e.m.f. and consequently the flux in the circuit TT' are nearly constant during normal operation.

Regarding the flux SS' the case is not quite so simple. In cutting the flux TT' a certain e.m.f. will be generated along the axis SS'. There will be sufficient current to produce a flux which will generate by transformer action approximately the same e.m.f. The flux in SS' will therefore be approximately proportional to the e.m.f. generated in SS' by the cutting of the flux TT'. If the motor is operating at synchronous speed the same e.m.f. will be generated whether it is due to the passage of the conductors through the flux or due to the change of the flux through the circuit of the conductors as in a transformer. In other words, it makes no difference whether the conductors are at rest and the flux in motion, or the flux is at rest and the conductors in motion. Hence at synchronous speed the transformer field TT' and the speed field SS' are approximately equal. At any other speed the e.m.f. generated by the motion of the conductors will be reduced or increased in proportion to the speed, and consequently the field SS' will be changed in the same proportion. If we let s equal the speed in per cent of synchronism and designate the two fields by T and S we have without serious error, $S=sT$.

In a similar way we have in the circuit TT' two e.m.fs., one due to the transformer action of the flux and the other to the cutting of the conductors through the flux SS'. At synchronism, these are as before approximately equal and opposite, the difference being just enough to establish the necessary current.

When load is applied to the motor, the rotor slows down somewhat. The speed field S is reduced to $S=sT$. The back e.m.f. in circuit TT' due to the cutting of the flux in the circuit SS' is then reduced to s^2T and this establishes more current through the circuit TT'. The current in SS' is slightly reduced. The increased current along TT' has a demagnetizing effect in the direction TT' and this immediately allows more current to flow through the stator coil to offset its action, and the flux remains at nearly its former value.

The current in the circuit TT' can exert no torque with the flux TT', and the same is true of the current and flux in the circuits SS'. Conversely, current in either of the two circuits is in the proper mechanical position to produce torque with the field corresponding to the

other circuit. The truth of these statements will readily appear from a consideration of Fig. 104 and 105. In these drawings the motor is represented as though it were provided with four projecting poles. The student must keep in mind that in the actual motor, the air gap is equal all around the rotor, and that the motor as illustrated is of the two-pole type.

With a current through the brushes SS' the distribution of current in the rotor would be as shown in Fig. 104. The circles represent currents directed toward the observer, and the crosses currents from him. It will be seen that the speed current and the transformer field will react to produce torque, and the same is true of the transformer

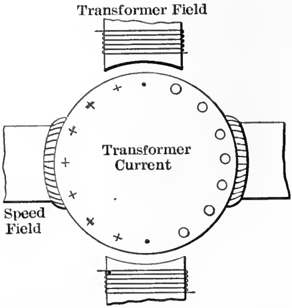


FIG. 104.—Relation of Current and Flux in Single-phase Motor.

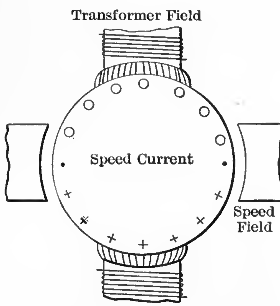


FIG. 105.—Relation of Current and Flux in Single-phase Motor.

current and the speed field. This is so since all the conductors lying in a field of given polarity carry currents in the same direction.

In addition to this fact, however, we must consider the effect of phase displacement. While the currents and fluxes, as shown in Fig. 104, are in the best mechanical position to produce torque, the actual average torque produced may be in either direction or may even be zero. This latter is in fact nearly the case with the speed current and the transformer field. The speed current is (except for the effect of hysteresis) in phase with the speed flux. This latter, as we have pointed out, differs 90 degrees in time-phase from the transformer flux, or the speed current is 90 degrees in time-phase from the transformer field. Hence at the time the current is a maximum the field is zero and

vice versa. Moreover, if we consider the moment when the flux is a maximum and the current is zero, it will be seen that during the previous quarter revolution the torque has been in one direction and that it will be in the opposite direction during the succeeding quarter revolution, since the current will be in the reverse direction. Hence it will be readily apparent that the average torque will be zero as far as the speed current and the transformer field are concerned.

Considering now Fig. 105, the conditions at no-load are nearly the same as those in the case of Fig. 104. The current in the stator coil being a magnetizing current is nearly in phase with the transformer flux. The rotor transformer current is nearly opposite in phase to the stator current. Hence it is 90 degrees out of phase with the speed flux, and as explained can exert no torque with it. This was of course to be expected, since at no-load the only torque required is enough to overcome the losses of the machine.

As the load on the motor is increased the current in the stator winding is increased by the addition of a component of current in phase with the applied e.m.f. This added current is consequently nearly 90 degrees different in phase from the transformer flux, or in phase with the speed flux. There will be produced in the rotor a current of the same m.m.f. as the added primary current and this will be in opposition to the primary current, and hence it will likewise be in the best phase to produce torque with the speed field. The total torque of the motor is then due to this *added* rotor current and to the speed field.

If we represent the torque by D, and the added rotor current by I_a we have, assuming suitable units,

$$\text{secondary input} = P_1 = I_a T.$$

In the same units, the torque in synchronous watts is given by the equation,

$$\text{Torque} = D = I_a S = sTI_a.$$

Also,

$$\text{output} = P_2 = sD = sTI_a.$$

Solving for s we get,

$$\frac{P_2}{P_1} = \frac{s^2TI_a}{TI_a} = s^2, \quad \text{or} \quad s = \sqrt{\frac{P_2}{P_1}};$$

or, the speed in per cent of synchronism is equal to the square root of the secondary efficiency.

The slip is given by $1-s$, and it will be readily seen that if the speed is near synchronism, the slip in percentage of the synchronous speed is equal to twice the percentage of rotor loss. Thus if the speed is 97 per cent, we have:

$$0.97=\sqrt{\frac{P_1}{P_2}}, \quad \text{or} \quad \frac{P_1}{P_2}=0.9409;$$

hence, we see that the slip is 3 per cent and the secondary loss is 5.91 per cent or nearly double the slip.

It must be kept in mind that the loss referred to is that due to the added component of the rotor current and is in fact a part only of the loss in the circuit TT'. In addition to this, there is a loss in both TT' and SS' due to the magnetizing components of the currents. The error in our conclusion is particularly apparent at synchronous speed when, according to this conclusion, we should have no rotor loss, while in reality there is a considerable loss due to the magnetizing currents. At a reasonably large load, the error becomes slight.

It will be remembered that in the case of the polyphase motor the slip is equal to the rotor loss. Thus in the single-phase motor, the rotor loss is approximately double that of the polyphase motor for the same slip.

There is another method of considering the single-phase induction motor, which, while apparently somewhat artificial, leads in some cases to very simple deductions. This method is based on the proposition that an alternating field, stationary in space, may be considered as being made up of two fields each of half the value of the stationary field, rotating in space in opposite directions with equal velocity. Thus in Fig. 106, if ϕ represents the instantaneous value of the alternating field, we may consider it as being made up of the two fields ϕ_a and ϕ_b rotating as shown. The vector sum of the two fields will at all times be equal to the value of the stationary field. Thus we may think of the rotor of the single-phase motor as being at the same time under the influence of two magnetic fields, rotating in opposite directions. The two fields are of equal strength and half the value of the actual stationary field.

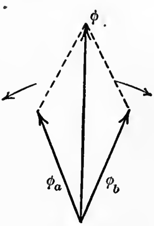


FIG. 106.—Resolution of a Stationary Flux into Two Revolving Fluxes.

With this in mind, we can very readily deduce the speed torque

curves of the single-phase motor. Thus in Fig. 107 are drawn several speed-torque curves of a wound-rotor polyphase induction motor. The curve A is that of the motor when no external resistors are connected in the rotor circuit. The curves B and C are for larger values of the external resistance. The derivation of these curves was fully explained on page 73. The curves are extended below the zero line to indicate the torque encountered in case the rotor is forced to rotate in the opposite direction to that in which it tends to turn. The curves A', B' and C' indicate the speed-torque curves of the same motor with the same resistors in the rotor circuit when it is operated in the opposite direction.

Suppose now that we have a single-phase motor operating at the speed represented by OK. From what has been said, it is evident that we may consider that the motor is subjected to the influence of two fields rotating in opposite directions. The one field will exert the torque KM, while the other field will oppose this torque with the torque KL. The net torque is the resultant of the two or LM. In Fig. 108 the length LM representing the torque at the speed OK, is laid off as shown. In the same way any number of points may be obtained. As was to be expected, the torque at standstill is zero.

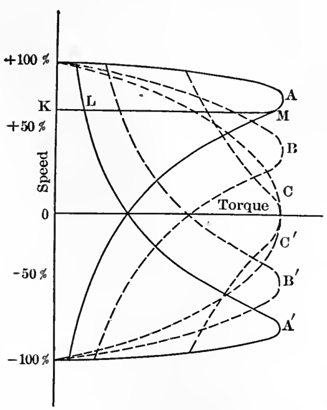


Fig. 107.—Speed Torque Curves of Polyphase Induction Motor.

In a similar manner, we can derive the curves for a single-phase motor with various values of resistance in the rotor circuit. Thus by combining the curves B and B' and C and C' of Fig. 107, we obtain the curves B and C of Fig. 108.

On examining the set of curves of Fig. 108 several new facts are at once apparent:

1. With a given magnetic field and a given rotor, the rated output can not be greater than half that of the same motor operated polyphase. This is apparent since if we are limited to a certain rotor loss the torque will be less than half as great, since each of the two component fields is of half the strength of the

corresponding polyphase field, and since there is a certain back torque due to the field which is rotating in the reverse direction.

2. The torque, and to a still greater extent the output, will be greater the less the secondary resistance. These facts are apparent from the curves. In this respect, the single-phase motor differs from the polyphase motor in which the maximum torque is independent of the secondary resistance.

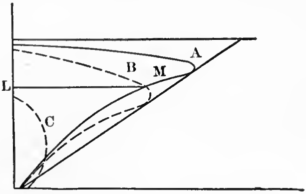


Fig. 108.—Speed Torque of Single-phase Induction Motor.

3. The plain single-phase induction motor has no starting torque. If, however, it be given a start in either direction, a small torque will be developed in that direction and if the load is not too great, the motor will accelerate to full speed. Small motors are sometimes started in this way, but this procedure is very unsatisfactory except in the case of motors of very small size, and for use in applications where the required starting torque is very small.

4. Although the maximum torque is reduced by increasing the resistance of the secondary by inserting resistors, the speed at which this torque is developed is also reduced. Hence the use of a moderate amount of resistance in the rotor will help the starting conditions, although the gain is by no means so great as in the case of the polyphase motor. In American practice, external resistors in the rotor circuit to assist in starting the motor are rarely or never used. Abroad, a number of motors have been built in which use is made of such an expedient.

CHAPTER XIV

THE SINGLE-PHASE INDUCTION REPULSION MOTOR

THE line of demarkation between the plain single-phase induction motor, as previously described, and the various types of motors usually described under the name of commutator type single-phase motors, is so slight that one is constantly tempted to overstep the line. In general, the commutator type single-phase motors have been developed with the idea in mind of using the motors in traction work. They have therefore had "series" characteristics, that is, if the load is removed the speed will increase almost without limit, and on the other hand, if the load be increased, the motor slows down very greatly and at the same time greatly increases its torque. The plain single-phase induction motor on the other hand, has a speed-torque characteristic of such a type that, as the load is increased, the speed decreases only to a slight extent. A motor the speed of which varies in this way is said to have a shunt characteristic. A motor of this type is desired for the greater number of applications, aside from railroad work and crane work.

The plain squirrel-cage single-phase induction motor has shunt characteristics and is therefore available for general purposes. It suffers, however, in comparison with the shunt-wound direct-current motor in having a power-factor less than one; in having, in its simplest form, no starting torque, and in having only a small starting torque even with special starting appliances, and in not being capable of having its speed adjusted to different values. Recently, considerable activity has been apparent among inventors in an attempt to overcome these deficiencies. These attempts in general involve the use of a commutator with its obvious disadvantages.

Considering the motor shown in Fig. 103, it is apparent that it has approximately the same characteristics as the plain squirrel-cage single-phase induction motor. In fact, we have seen that all the characteristics of the squirrel-cage machine can be derived from a consideration of this form of motor. It likewise has no starting torque,

and since it has no advantage and suffers from the presence of the commutator, it is clearly not adapted to general use. By a slight shift of the brushes it is however possible to give the motor a good starting torque.

To understand this, consider what would happen in the motor of Fig. 103, if of the two circuits between the two sets of brushes only one, say TT', were closed. This circuit is in the best position relative to the stator winding to have current produced in it, and consequently if the circuit TT' alone were closed and current were applied to the motor at standstill, there would be a large current established in the rotor circuit. It is, however, apparent that the current in the rotor would be so related to the primary flux that the resultant torque would be zero. If, on the other hand, only the circuit SS' were closed, no current would be produced in it at standstill, and consequently there would be no torque. It is however true, that if there were current in SS' it would be in the best possible position to produce torque in connection with the primary flux.

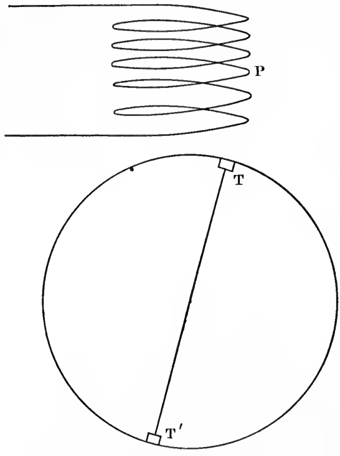


Fig. 109.—Connections of Repulsion Motor.

If now one of the circuits between the two brushes be closed and the brushes be set in any position except in line with or at right angles to the line of the stator coil, as shown in Fig. 109, there will be a resultant torque. A motor of this type with only the one set of brushes on the commutator is known as a repulsion motor. Such motors are used to some extent in railroad work. They have, however, series characteristics, that is the speed increases almost without limit as the load is decreased and hence are not suited to the class of service we are at present considering. This principle is, however, utilized in a number of successful single-phase induction motors as a means of starting. In this case, some form of device, usually a couple of rotating weights operating by

centrifugal action, is used to remove the brushes from the commutator at the proper speed, and at the same time, short-circuit all the segments of the commutator. In this way, the machine is transformed into an ordinary single-phase induction motor with a rotor which is practically equivalent to a squirrel-cage rotor.

Returning now to the type of motor represented in Fig. 103, but giving the brushes a small lead in the one direction or the other, as shown in Fig. 110, it will be apparent that with either set of brushes alone, there would be a starting torque. With both sets of brushes in use, there will be a resultant torque, due to the combined action of the two. With the angle as shown, the action of the brushes TT' will be far greater than that of SS', and the direction of rotation would be clockwise. If the shift of the brushes had been in the opposite direction, the direction of rotation would also have been reversed.

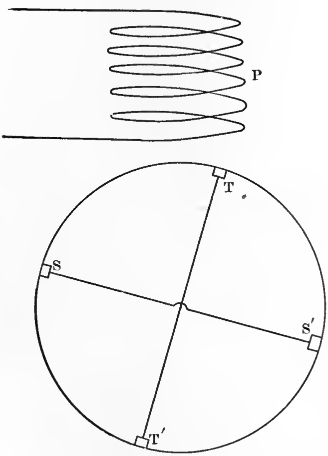


Fig. 110.—Single-Phase Induction Motor with Brushes Displaced to give Starting Torque.

This motor, unlike the repulsion motor, does not tend to increase in speed indefinitely but approaches a limiting speed somewhat near to the synchronous speed. As we have seen, if the brushes are in line with the stator coil, the machine will act in all respects like an induction motor and will have a maximum speed somewhat below synchronism. This is not the case with the motor represented in Fig. 110. The no-load speed of the motor will be somewhat above synchronous speed, while the full-load speed will in general be below synchronism.

So far, we have arrived at a single-phase induction motor having shunt characteristics, and possessing a fairly large starting torque. With this construction it is, however, possible at a slight additional expense, to improve the power-factor of the motor. This is done by

adding an additional coil wound in the same slots as the main winding and connected to the brushes SS'. The connections are shown in Fig. 111.

Instead of using a separate compensating coil, it is evident that a portion of the main stator winding may be used instead. This connection is shown in Fig. 112.

A comparison of this connection with that of the polyphase motor shown in Fig. 49, will show that the connections are essentially the same.

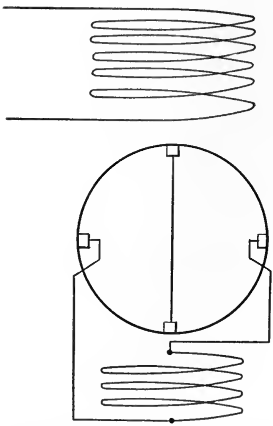


FIG 111.—Single-phase, Compensated Induction Motor.

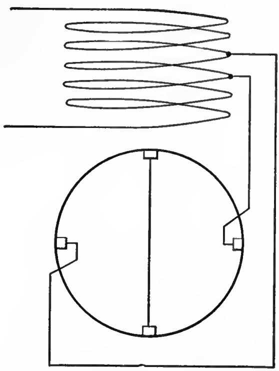


FIG. 112.—Single-phase Compensated Induction Motor.

The action of the compensating coil and brushes C in improving the power-factor is practically the same as the action of the auxiliary brushes in the case of the polyphase motor. It will be remembered that there are in the circuit SS' two approximately equal and opposite e.m.fs., the transformer e.m.f. and the speed e.m.f., both of these are in phase with the flux in the circuit TT' and consequently in quadrature with the primary e.m.f. The current in the circuit SS' is in quadrature with both of these e.m.fs., and of course in phase with the flux in the same circuit. The e.m.f. applied from the compensating coil is therefore approximately in phase with the current and flux, and at right angles to the two e.m.fs. already present in the SS' circuit.

We can perhaps best appreciate the effect of this added e.m.f. by considering that the action as already described goes on as before, but that since we have now added a new e.m.f. in the speed circuit, there will be produced a new flux so as to generate by transformer action a new e.m.f. approximately equal and opposite to that which we have introduced from the compensating coil. This new flux is at right angles in time-phase to the flux formerly present, but in the same position as regards the core. We may call this added flux the compensating flux.

The compensating flux, being at right angles to the speed flux, has no effect on the torque of the motor, and hence has little or no effect on the speed. It has, however, the effect of introducing a new speed e.m.f. in the transformer axis TT'. This new e.m.f., which we may call by analogy the compensating e.m.f., being at right angles to the speed and transformer e.m.f.s in the transformer axis, tends to combine with their resultant to form a new resultant e.m.f. We may, however, as before consider the effect of this e.m.f. by itself. Both the speed and the transformer e.m.f. in the TT' circuit are in phase with and in phase opposition to the primary counter generated e.m.f. and nearly so with the primary applied e.m.f. The compensating e.m.f. is therefore nearly at right angles to the primary applied e.m.f. that was present before its introduction. The compensating e.m.f., however, tends to set up a current and a flux nearly at right angles to itself, the flux being of such a value as to generate by transformer action, approximately the same e.m.f. as the compensating e.m.f. This flux in turn generates a new e.m.f. in the stator approximately at right angles to the transformer voltage which would be present without it. The total counter e.m.f. of the primary circuit is the resultant of these two e.m.fs. and consequently its phase can be changed so as to lead the primary current by a greater angle, or if is applied in the opposite direction, the e.m.f. may be made to lag behind the primary current or to be in phase with it; that is, the motor may be made to take a current in phase with the applied e.m.f. or even a leading current.

In Fig. 113 are shown the characteristic curves ot a small General Electric motor of this type. The connections are as shown in Fig. 112. It will be seen that the power-factor is high for all loads. At no-load, the compensating brushes and the main brushes each carry current. These two currents are in quadrature and serve to furnish the magnetizing currents in the two axes of the motor. The stator current is comparatively small, and serves principally to furnish by transforma-

tion the current required for the compensating brushes. As the load is increased, the current in the primary coil is increased by a component

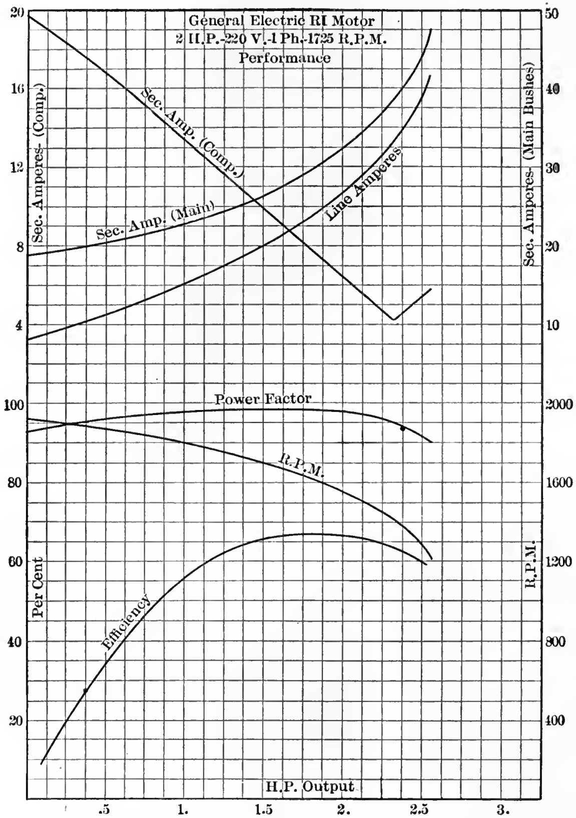


FIG. 113.—Characteristic Curves of General Electric Type R.I. Motor.

in phase with the e.m.f., and the current in the main brushes is correspondingly increased.

The current in the compensating brushes on the other hand constantly decreases, reaches nearly zero and finally increases. At the point of smallest current through the compensating winding, the brushes SS' might be removed, and the conditions of running would obviously not be changed. In other words, the motor at this particular load is operating practically as a repulsion motor. At all other loads, it is forced to operate at either a higher or a lower speed than would be the case as a repulsion motor.

Speed-control of Single-phase Motors

A single-phase motor of this type admits of efficient speed control through at least a limited range of speed. The principles upon which this is based are similar to those explained in connection with the polyphase motor. The single-phase motor however admits of some extension of these methods, and will therefore be described separately.

In the following, the diagrams of the motors will be drawn without the compensating coil for raising the power-factor. It will, however, be understood that in practice such a coil would usually be provided in addition to the windings as shown.

There are in general two methods of speed control, and these may be called by analogy with the shunt-wound direct-current motor, the armature control and field control. To change the speed by the first method, an e.m.f. of the primary frequency and of the same phase as the applied e.m.f. must be introduced into the TT' circuit. To employ the method of field control an e.m.f. differing in phase 90 degrees from the primary e.m.f. must be introduced into the SS' circuit.

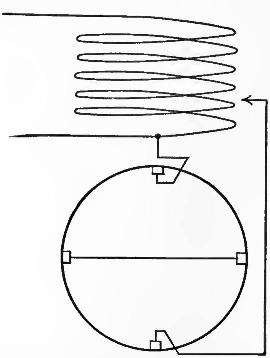


Fig. 114.—Single-phase Motor with Armature Speed Control.

In Figs. 114 and 115 are shown two examples of speed control by the first method. In Fig. 114 taps are brought out from the primary winding and the e.m.f. between these points is introduced into the TT' circuit. In Fig. 115 an auto-transformer is provided to give the required voltage. Instead of an auto-transformer

an ordinary transformer with primary and secondary coils might have been provided. It will be remembered that we have in the TT' circuit two voltages, one due to the transformer action of the primary flux, the other due to the cutting of the flux of the speed field. With the brushes TT' short-circuited these two are approximately equal and opposite and at right angles to the primary flux. They are consequently in phase and phase opposition respectively to the primary applied e.m.f. If we add another e.m.f. in phase with the primary voltage, we destroy this equality and there will be a large current until a balance is re-established. This current

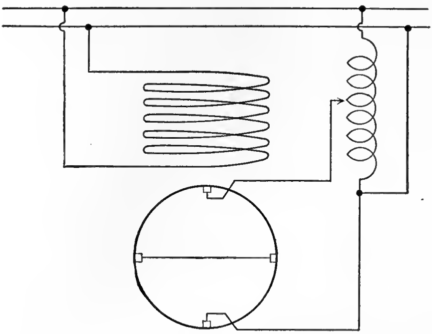


FIG. 115.—Single-phase Motor with Armature Speed Control.

will be nearly in phase with the primary e.m.f. The flux in the SS' axis is in the same phase and consequently the current in the TT' axis is in the best position to produce torque with the flux in the SS' axis. As a consequence, the motor either speeds up or slows down until the current in TT' is reduced to that value which with the flux in the SS' axis will give sufficient torque to overcome the torque due to the load, the friction and losses of the machine.

The action while similar is not exactly the same as that in the shunt-wound direct-current motor. If, for example, the motor speeds up, the speed e.m.f. in the SS' axis is increased. This immediately causes a greater flux through the SS' axis to provide the correspondingly greater transformer e.m.f. in this circuit. This, in turn, means that the speed of the motor does not have to increase so much as would otherwise be the case in order that the increased speed voltage in the TT'

axis may be generated. Hence, the increase in speed in not in the ratio of $\frac{E+E_1}{E}$ but in the ratio of $\sqrt{\frac{E+E_1}{E}}$. A similar argument will apply in the case of reduced speed, the speed field in this case being weakened. The speed may also be lowered, as shown in Fig. 116, by inserting resistance in the TT' circuit. This has the effect of weakening the current in this circuit until the motor slows down sufficiently to make the difference of the two e.m.fs. great enough to establish the requisite current. Using this method, the speed for varying loads will not be constant, and there will be a large loss of power in the regulating resistance. The arrangement is similar to that of a shunt-wound direct-current motor the speed of which is controlled by adjusting the resistance of the armature circuit. Obviously, only a decrease of speed can be obtained by this means.

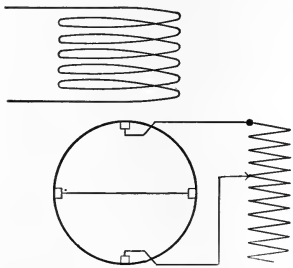


Fig. 116.—Speed Control of Single-phase Motor by Armature Resistance.

In Figs. 117 and 118 are shown two of the many possible methods of speed control by field variation. If an e.m.f. in phase with the line voltage be introduced into the brushes SS', it will have little or no effect on the speed. This is in fact exactly what we do to improve the power-factor by compensation, and as was shown in the discussion of compensation, the speed is not materially changed.

In the connection as made in Fig. 117, if the windings are so connected that the ampere-turns of the rotor oppose those of the coil in the speed axis, the flux along this axis will be lessened. If connected in the opposite manner it will of course be strengthened. If the former,

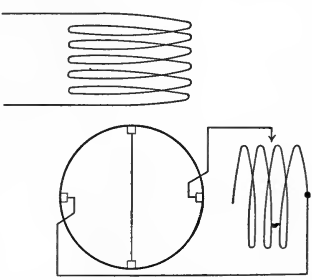


Fig. 117.—Speed Adjustment of Single-phase Motor by Field Control.

the speed e.m.f. in the transformer axis will be lessened and the motor will operate at a higher speed in order to make this e.m.f. approximately equal to the transformer e.m.f. If on the other hand the flux be increased the motor of course operates at a higher speed.

Another way of obtaining the same effect is shown in Fig. 118. A reactance is connected in the circuit of the speed brushes, and taps are brought out so that the value of the reactance can be readily varied. The action may be looked at in two ways. We may consider that we are inserting an e.m.f. in the circuit of the speed brushes. This e.m.f.

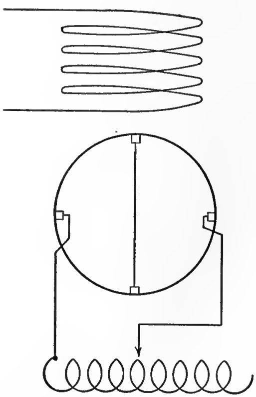


FIG. 118.—Single-phase Motor with Speed Control by Means of Reactance in the Field Circuit.

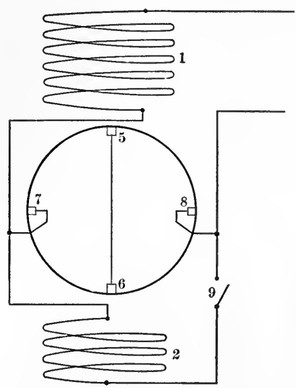


FIG. 119.—Connections of Wagner Type B.K. Motor.

is the back e.m.f. due to the reactance. It is, of course, 90 degrees in phase from the current or in phase with the speed and transformer e.m.fs. in the axis. This would, as we have shown, have the effect of reducing the flux in this axis and consequently of increasing the speed of the motor.

The other way of looking at the matter is to consider that we have transfered a portion of the flux to a location where it is still available for generating the transformer e.m.f. in the speed axis, but is not available for generating speed e.m.f. In order therefore to generate the proper speed e.m.f. in the TT' axis due to cutting the flux in the SS' axis, the motor will be obliged to rotate at a greater speed. This

method is obviously available only for increasing the speed, and not for reducing it.

Another motor operating on the same general principles is the Wagner type BK. This motor is provided with a commutated winding of the same nature as a direct-current winding, but has in addition a squirrel-cage winding of the usual type. The electrical connections are shown in Fig. 119; and a section of a slot showing both the commutated winding and the squirrel-cage is given in Fig. 120.

During starting, the switch "9" is open, and the winding "2" carries no current. Disregarding for the moment the squirrel-cage, the connections of the motor are the same as those of the so-called compensated repulsion motor. Such motors are used to some extent as single-phase railway motors, and of course have series characteristics. The motor therefore starts with excellent torque. The torque due to this winding, however, decreases rapidly as the speed increases, as is the case with all series motors.

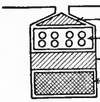


FIG. 120.—Slot of Wagner Type B.K Motor..

In addition to this torque there is a torque due to the squirrel-cage. This latter torque is zero at start, rapidly increases as the speed increases, reaching a maximum at about 4 or 5 per cent slip, and becomes zero at synchronism. Above synchronism the torque due to the squirrel-cage reverses. The general shape of this curve is as shown in Fig. 103. The torque curve of the motor is the resultant of these two curves. There is a peak in the curve at zero speed, a slight lowering for intermediate speeds, and another and slightly higher peak at about 90 per cent of synchronous speed. With a starting torque of 160 per cent, the minimum torque during acceleration is stated to be 133 per cent.

The squirrel-cage winding is intended to be comparatively inactive at low speeds, and to carry the principal part of the current at speeds near synchronism. This object is accomplished by placing the squirrel-cage winding in the bottom of the slots and using the steel bars shown in Fig. 120, between the two windings. The use of these steel bars causes the local leakage reactance of the squirrel-cage to be high. At start, the frequency of the currents in the squirrel-cage is the same as

the line frequency. On account of the high reactance, the secondary current produced is small, and the commutated winding carries the larger part of the current.

When the motor has nearly reached synchronism, a centrifugal switch operates to close the circuit through the switch " 9." The motor then operates as far as the commutated winding is concerned in exactly the same manner as the ordinary compensated single-phase commutator type motor. It will be noted that with the switch " 9 " closed, the winding " 2 " becomes a part of the stator winding. The squirrel-cage winding, however, takes its share of the current and the result is that the slip of the motor under load is very small.

The squirrel-cage is also of advantage in that is prevents the possibility of the motor running away. Without it, if one of the brushes " 7 " or " 8 " should cease to make contact with the commutator and the load on the motor were light, the motor might speed up greatly and damage itself. With the squirrel-cage winding this is impossible. Even though all of the brushes should cease to make contact with the commutator, the motor would operate as before, and no change would be apparent to the eye. The capacity would however be reduced, and the power-factor lowered. The motor would, however, continue to operate indefinitely without injury.

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